
Cuaderno de notas de trabajo

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Cuaderno 11

$$1) H_{ij} = \frac{M[2\vec{V}_i \cdot \vec{V}_j - \Delta_{ij}]}{(\vec{r} \cdot \vec{V})}$$

$$2) \vec{r} = \vec{\xi} - \vec{R}$$

$$3) \frac{d}{dS}$$

$$4) \frac{d\vec{V}_i}{dS} = A_i; \quad \frac{d\vec{V}_j}{dS} = A_j$$

$$5) \frac{d\vec{r}}{dS} = -\vec{V}$$

$$6) \frac{d}{dS} (\vec{r} \cdot \vec{V}) = -1 + (\vec{r} \cdot \vec{A})$$

$$7) \frac{d}{dS} H_{ij} = \left[\frac{M[2\vec{V}_i \cdot A_j + 2A_i \cdot \vec{V}_j]}{(\vec{r} \cdot \vec{V})} - \frac{M[2\vec{V}_i \cdot \vec{V}_j - \Delta_{ij}]}{(\vec{r} \cdot \vec{V})^2} [-1 + \vec{r} \cdot \vec{A}] \right]$$

Incremento del campo H_{ij}
debido al ~~la~~ cuadrivector aceleración
de un punto masa.

Δ — incremento parcial

(se descuenta — o no se toma en cuenta — el incremento del campo debido al vector velocidad $\vec{V}$),

$$\Delta H_{ij} = \left[\frac{M [2V_i A_j + 2A_i V_j]}{(\vec{r} \cdot \vec{V})} \Delta S - \frac{M [2V_i V_j - \Delta_{ij}]}{(\vec{r} \cdot \vec{V})^2} (\vec{r} \cdot \hat{A}) \Delta S \right]$$

$$\Delta^{ij} \Delta H_{ij} = - \frac{M [2 - \Delta^{ij} \Delta_{ij}]}{(\vec{r} \cdot \vec{V})^2} (\vec{r} \cdot \hat{A}) \Delta S$$

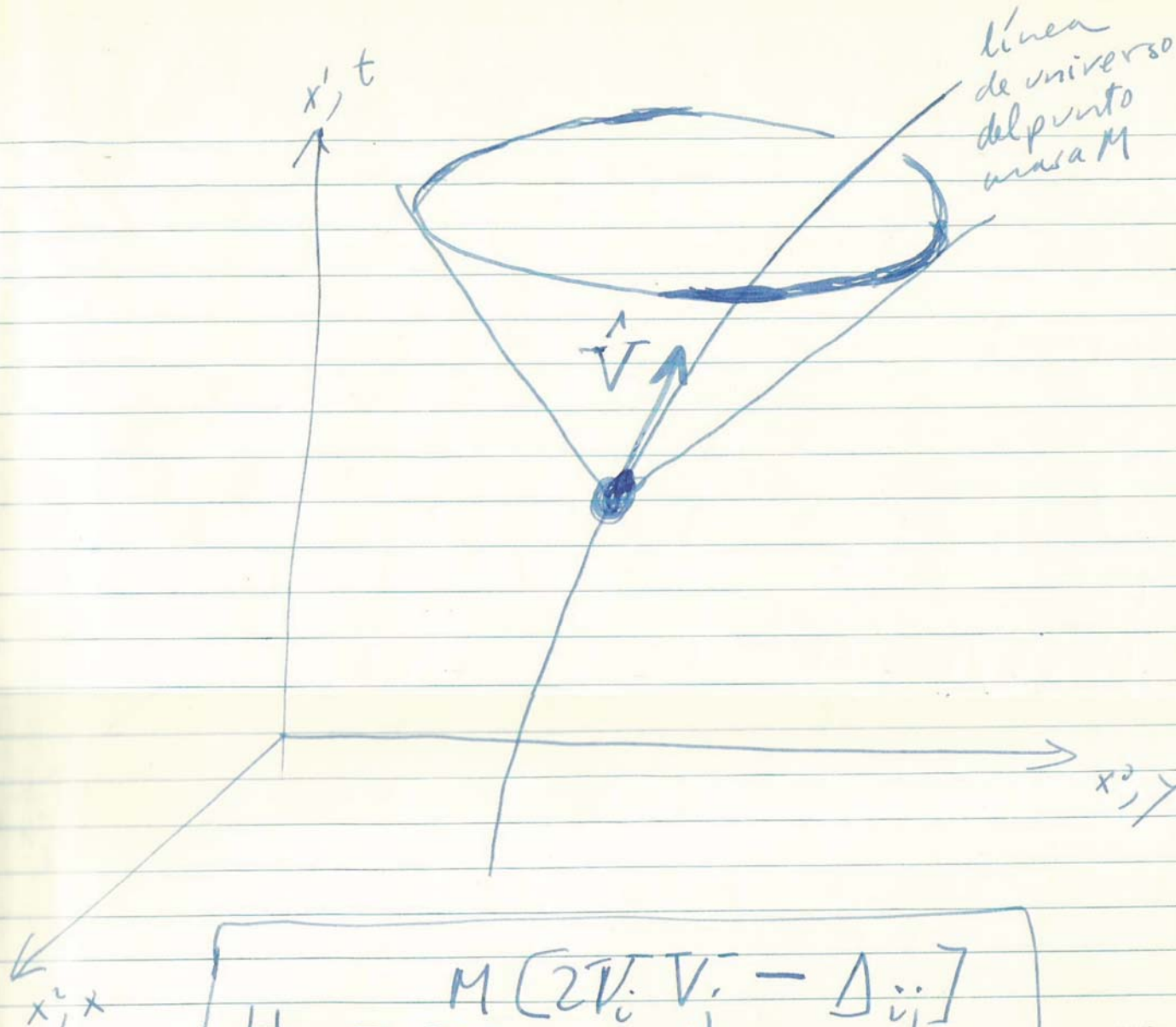
~~Δ_{ij}~~

$$\Delta^{\dot{j}} V_i A_j = 0$$

$$\Delta^{\dot{j}} V_i V_j = 1$$

$$\Delta^{\dot{j}} \Delta_{ij} = 4$$

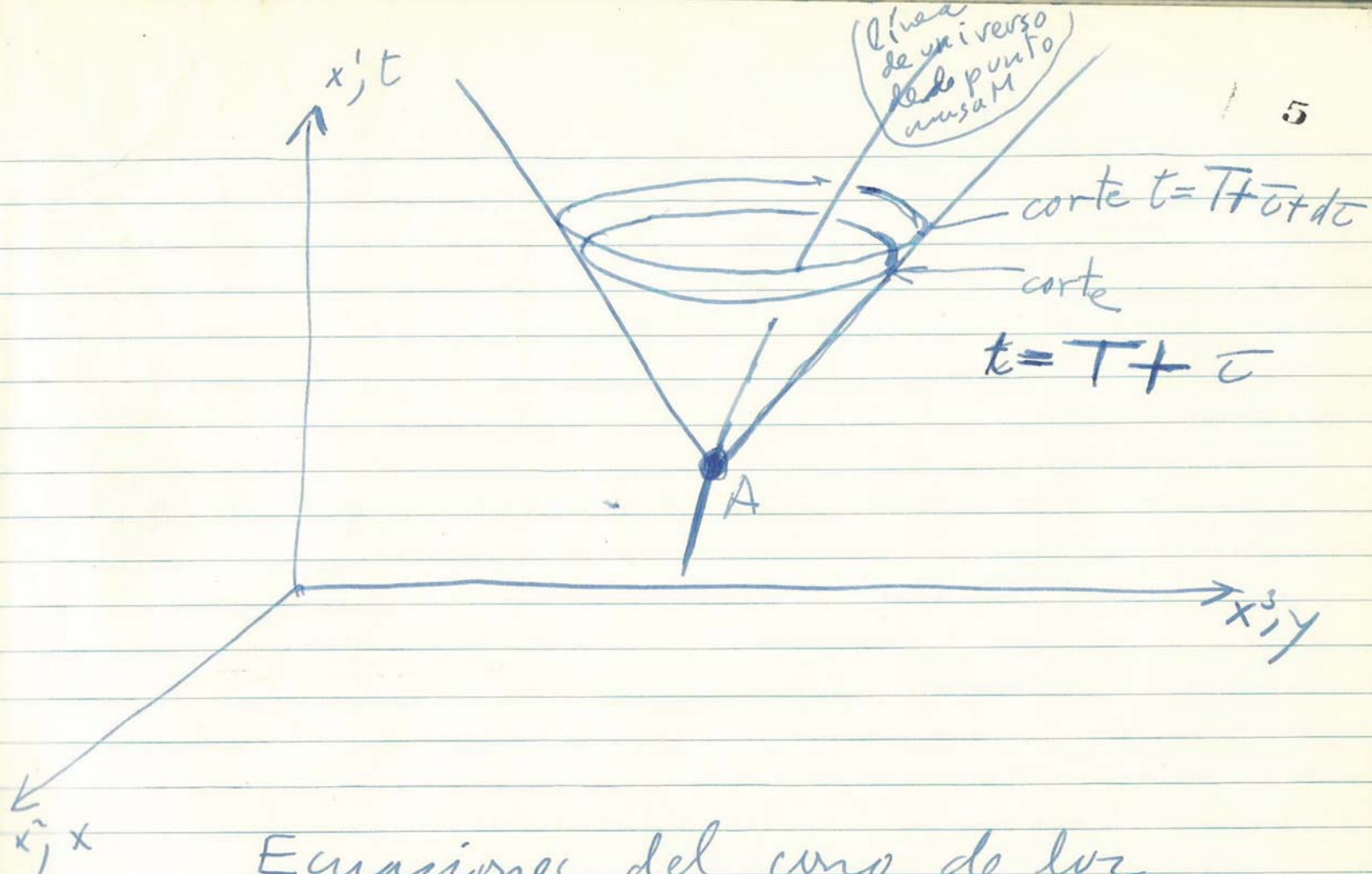
$$\Delta^{\dot{j}} \Delta H_{ij} = \frac{2H(\mathbf{A} \cdot \hat{\mathbf{A}})}{(\mathbf{A} \cdot \hat{\mathbf{V}})^2} \Delta S$$



$$H_{ij} = \frac{M [2V_i V_j - \Delta_{ij}]}{(\hat{r} \cdot \hat{V})}$$

$$\Delta^{ij} H_{ij} = \frac{M [2 - 4]}{(\hat{r} \cdot \hat{V})}$$

$$\Delta^{ij} H_{ij} = - \frac{2M}{(\hat{r} \cdot \hat{V})}$$



Ecuaciones del cono de luz con vértice en A.

$$t = T + \bar{t};$$

$$x = \bar{X} + \bar{r} \operatorname{sen} \theta \cos \varphi;$$

$$y = \bar{Y} + \bar{r} \operatorname{sen} \theta \operatorname{sen} \varphi;$$

$$z = \bar{Z} + \bar{r} \cos \theta.$$

$$(t - T)^2 - (x - \bar{X})^2 - (y - \bar{Y})^2 - (z - \bar{Z})^2 = 0$$

Elemento de hipervolumen

Cuatro cuadrivectores elementales:

$d\hat{r}$ cuando sólo varía S ;

$\delta\hat{r}$ cuando sólo varía τ ;

~~$\Delta\hat{r}$~~ $\Delta\hat{r}$ cuando sólo varía θ ;

~~$\nabla\hat{r}$~~ $\nabla\hat{r}$ cuando sólo varía φ .

$$d\hat{r} = \hat{V} dS$$

$$\delta\hat{r} = (d\tau, d\tau \sin\theta \cos\varphi, d\tau \sin\theta \sin\varphi, d\tau \cos\theta)$$

~~$\Delta\hat{r}$~~

$$\Delta\hat{r} = (0, \tau \cos\theta \cos\varphi d\theta, \tau \cos\theta \sin\varphi d\theta, -\tau \sin\theta d\theta)$$

$$\nabla\hat{r} = (0, -\tau \sin\theta \sin\varphi d\varphi, \tau \sin\theta \cos\varphi d\varphi, 0)$$

El elemento de hipervolumen

~~dH~~ dH

$$dH =$$

V^1	V^2	V^3	V^4	$dS/d\theta/d\varphi$
1	$\sin\theta \cos\varphi$	$\sin\theta \sin\varphi$	$\cos\theta$	
$\tau \cos\theta \cos\varphi$ 0	0 $\tau \cos\theta \cos\varphi$	$\tau \cos\theta \sin\varphi$	$-\tau \sin\theta$	
0	$-\tau \sin\theta \sin\varphi$	$\tau \sin\theta \cos\varphi$	0	

Coefficiente de V^1

$$+ \cos\theta (\tau^2 \sin\theta \cos\theta \cos^2\varphi + \tau^2 \sin\theta \cos\theta \sin^2\varphi) \\ + \tau \sin\theta (\tau \sin^2\theta \cos^2\varphi + \tau \sin^2\theta \sin^2\varphi)$$

$$= \tau^2 \sin\theta \cos^2\theta + \tau^2 \sin^3\theta$$

$$= \tau^2 \sin\theta$$

$$\boxed{\text{Coeficiente de } V^1 = \tau^2 \sin\theta}$$

Coefficiente de V^2

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$$- \begin{vmatrix} 1 & \sin \theta \sin \varphi & \cos \theta \\ 0 & \tau \cos \theta \sin \varphi & -\tau \sin \theta \\ 0 & \tau \sin \theta \cos \varphi & 0 \end{vmatrix}$$

$$- \tau^2 \sin^2 \theta \cos \varphi$$

$$\text{Coefficiente de } V^2 = -\tau^2 \sin^2 \theta \cos \varphi$$

Coefficiente de V^3

$$+ \begin{vmatrix} 1 & \sin \theta \cos \varphi & \cos \theta \\ 0 & \tau \cos \theta \cos \varphi & -\tau \sin \theta \\ 0 & -\tau \sin \theta \sin \varphi & 0 \end{vmatrix}$$

$$= -\tau^2 \sin^2 \theta \sin \varphi$$

$$\text{Coefficiente de } V^3 = -\tau^2 \sin^2 \theta \sin \varphi$$

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coeficiente de V^{-4}

$$- \begin{vmatrix} 1 & \tau \cos \theta \cos \varphi & \tau \cos \theta \sin \varphi \\ 0 & -\tau \sin \theta \cos \varphi & \tau \sin \theta \sin \varphi \\ 0 & \tau \cos \theta \cos \varphi & -\tau \cos \theta \sin \varphi \end{vmatrix}$$

$$- [\tau^2 \sin \theta \cos \theta \cos^2 \varphi + \tau^2 \sin \theta \cos \theta \sin^2 \varphi]$$

$$- \tau^2 \sin \theta \cos \theta$$

Coeficiente de $V^4 = -\tau^2 \sin \theta \cos \theta$

$$dH = \begin{bmatrix} \tau^2 \sin \theta V^1 \\ -\tau^2 \sin^2 \theta \cos \varphi V^2 \\ -\tau^2 \sin^2 \theta \sin \varphi V^3 \\ -\tau^2 \sin \theta \cos \theta V^4 \end{bmatrix} d\varphi d\tau d\theta d\varphi$$

$$\Delta^{ij} H_{ij} = - \frac{2M}{(\hat{r} \cdot \hat{V})}$$

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$$\hat{r} = [\tau, \tau \sin \theta \cos \varphi, \tau \sin \theta \sin \varphi, \tau \cos \theta]$$

$$\hat{V} = [V^1, V^2, V^3, V^4]$$

$$(\hat{r} \cdot \hat{V}) = \left[\tau V^1 + \tau V^2 \sin \theta \cos \varphi + \tau V^3 \sin \theta \sin \varphi + \tau V^4 \cos \theta \right]$$

$$\frac{[\Delta^{ij} H_{ij}]^2}{dk} dH =$$

$$\Delta H_{ij}^2 = \frac{r^2 [V_{\text{sen } \theta}^1 - V_{\text{sen } \theta}^2 \cos \theta - V_{\text{sen } \theta}^3 \cos \theta - V_{\text{sen } \theta}^4 \cos \theta]^2}{r^2 [V^1 - V^2 \cos \theta - V^3 \cos \theta - V^4 \cos \theta]^2}$$

→ divide both sides

$$[\Delta H_{ij}]^2 = \frac{\text{sen } \theta \cancel{dS} d\theta d\varphi}{V^1 - V^2 \cos \theta - V^3 \cos \theta - V^4 \cos \theta}$$

Potencial Newtoniano de una
esfera de radio β y de densidad
constante ρ .

Para $r > \beta$ $V_e = -\frac{M}{r}$ ext.

M — masa total de la esfera

$$\rho = \frac{M}{\frac{4}{3}\pi\beta^3}$$

Para $r < \beta$ $\left| \frac{\frac{4}{3}\pi r^3 \cdot M}{\frac{4}{3}\pi\beta^3} = \frac{Mr^3}{\beta^3} \right.$

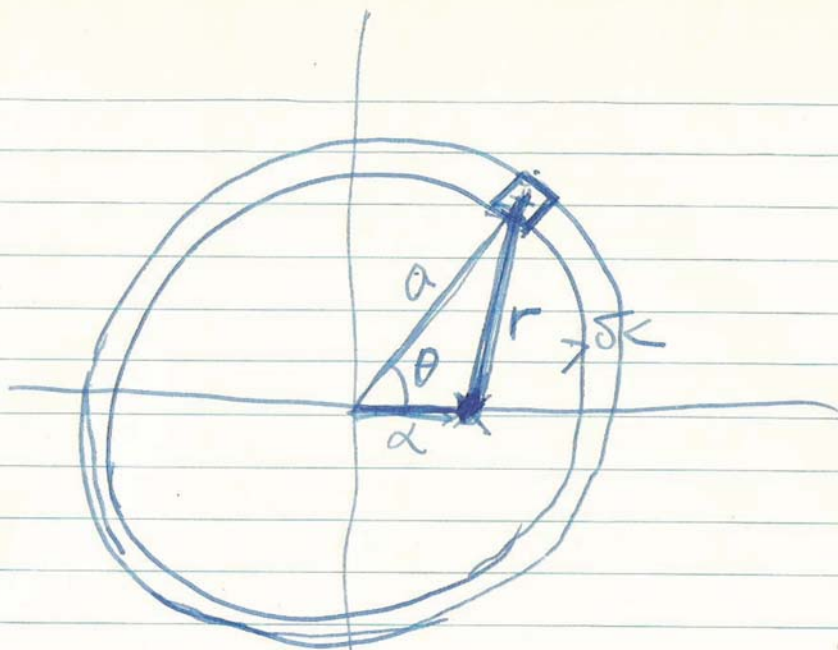
~~πr^2~~

$$V_i = -\frac{\frac{Mr^3}{\beta^3}}{r} = -\frac{Mr^2}{\beta^3}$$

interior

$$V_{int.} = -\frac{Mr^2}{\beta^3}$$

$$V_{ext.} = -\frac{M}{r}$$



$$r = \sqrt{a^2 + \alpha^2 - 2a\alpha \cos \theta}$$

$$\int \frac{a d\theta \cdot a \sin \theta \cdot 2\pi \delta}{\sqrt{a^2 + \alpha^2 - 2a\alpha \cos \theta}}$$

$$2\pi a^2 \delta \int_0^\pi \frac{\sin \theta d\theta}{\sqrt{a^2 + \alpha^2 - 2a\alpha \cos \theta}}$$

$$a^2 + \alpha^2 - 2a\alpha \cos \theta = v^2$$

$$2a\alpha \sin \theta d\theta = 2v dv$$

$$2\pi a^2 \delta \int_0^\pi \frac{2v dv}{2a\alpha v}$$

$$V = \frac{2\pi a^x \delta \rho}{a^x} \int_{a-x}^{a+x} du$$

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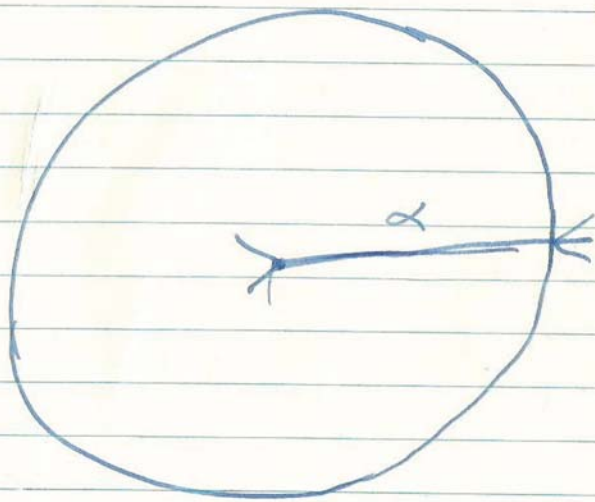
$$V = \frac{2\pi \cdot \delta \cdot \rho}{x} \cdot 2x$$

$$V = 4\pi a \delta \cdot \rho$$

~~$$V = \frac{4\pi a^2 \delta \cdot \rho}{a} = \frac{4\pi a \delta \cdot \rho}{1}$$~~

$$\int_{r_1}^{r_2} 4\pi r dr \cdot \rho = \frac{4\pi \rho}{2} (r_2^2 - r_1^2)$$

Esfera de radio α y de densidad constante ρ .
Masa total M



$$\rho = \frac{M}{\frac{4}{3}\pi\alpha^3} = \frac{3M}{4\pi\alpha^3}$$

$$\boxed{\rho = \frac{3M}{4\pi\alpha^3}}$$

Potencial Newtoniano dentro de la esfera.

$$V = -\frac{4}{3}\pi r^3 \cdot \frac{3M}{4\pi\alpha^3 \cdot r}$$

$$\boxed{V = -\frac{Mr^2}{\alpha^3}} \quad \text{Para } r \leq \alpha$$

Potencial Newtoniano fuera de la esfera 16

$$V = -\frac{M}{r} \quad \text{Para } r \geq \alpha$$

La integral de volumen de $V d\tau$ ← (de volumen) sobre todo el espacio

$$\int_{\text{Todo el espacio}} V d\tau = \left[\int_0^\alpha \left[-\frac{M r^2}{\alpha^3} 4\pi r^2 dr \right] + \int_\alpha^\infty \left[-\frac{M}{r} 4\pi r^2 dr \right] \right]$$

$$\int_{\text{el espacio}} V d\tau = - \left[\frac{4\pi}{\alpha^3} \frac{r^5}{5} \right]_0^\alpha + \frac{M}{r}$$

$$\int V^n d\vec{c} = \int_0^{\alpha} \left[(-)^n \frac{M^n r^{2n}}{\alpha^{3n}} \right] 4\pi r^2 dr$$

$$+ \int_{\alpha}^{\infty} (-)^n \frac{M^n}{r^n} 4\pi r^2 dr$$

Todo el espacio

$$4\pi (-)^n \frac{M^n}{\alpha^{3n}} \left[\frac{r^{2n+3}}{2n+3} \right]_0^{\alpha} + \frac{4\pi (-)^n M^n}{-n+3} \left[\frac{r^{-n+3}}{-n+3} \right]_{\alpha}^{\infty}$$

$$4\pi (-)^n M^n \left[\frac{\alpha^{2n+3}}{2n+3} - \frac{\alpha^{-n+3}}{3-n} \right]$$

$$2n+3 = 3-n$$

$$3n = 0$$

$$n = 0$$

M — masa de un punto masa.

h_{ij} — campo gravitacional en el que se mueve M .

A_i — cuadrivector aceleración de Minkowski que el campo imparte a M .

$$A_i = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) V^j V^k.$$

Tensor de energía y momento del punto masa M

~~E_{ij}~~ $T_{ij} = M V_i V_j$

~~δ~~

Incremento de T_{ij} en el recorrido del arco dS de la línea de universo de M .

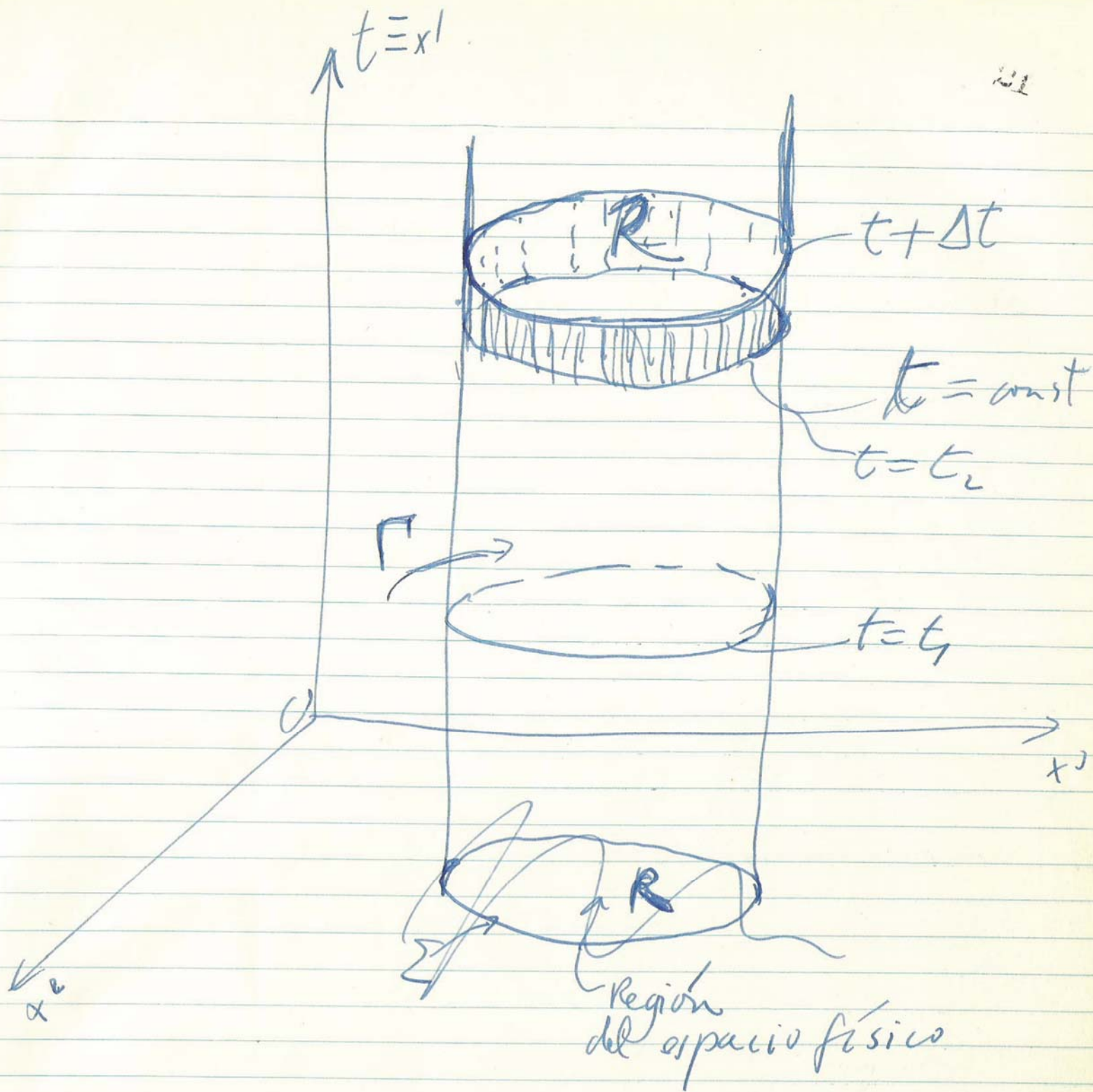
$$dT_{ij} = [M(V_i A_j + A_i V_j)] dS$$

$$A_i = \frac{\partial h_{il}}{\partial x^k} V^l V^k - \frac{\partial h_{ik}}{\partial x^i} V^l V^k$$

$$A_j = \frac{\partial h_{lj}}{\partial x^k} V^l V^k - \frac{\partial h_{lk}}{\partial x^j} V^l V^k$$

$$A_i V_j = \frac{\partial h_{il}}{\partial x^k} V^l V^k V_j - \frac{\partial h_{ik}}{\partial x^i} V^l V^k V_j$$

$$A_j V_i = \frac{\partial h_{lj}}{\partial x^k} V^l V^k V_i - \frac{\partial h_{lk}}{\partial x^j} V^l V^k V_i$$



Observador inercial O

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④ - - - - - región cuatridimensional

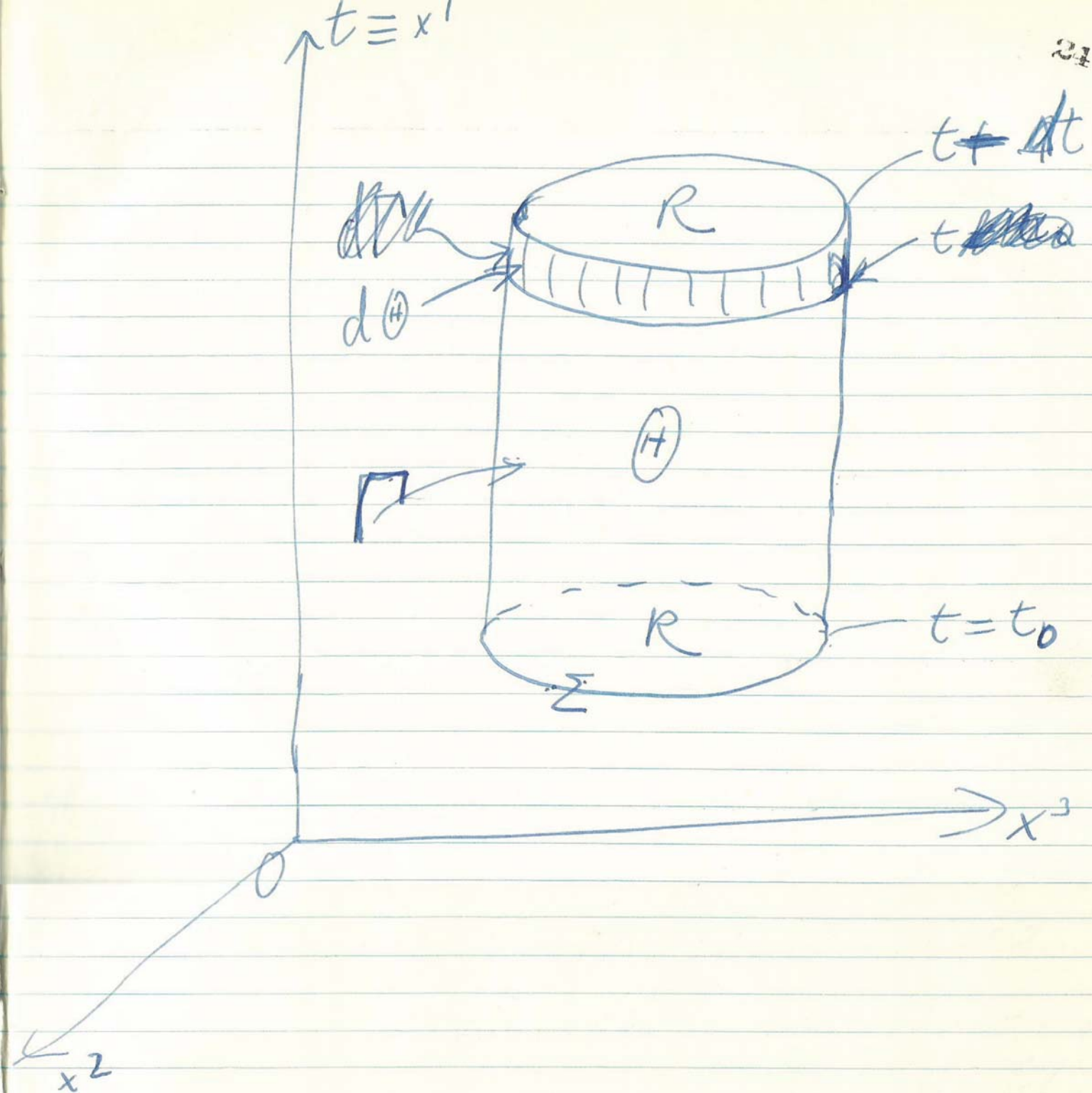
1) R - - - - - región del espacio físico;
esa región es permanente.

2) Σ - - - - - Es la superficie que
limita a la región R .

3) R y Σ están fijas para el
observador O .

4) Las líneas de universo de
los puntos de Σ forman
la hipersuperficie Γ .

Esas líneas de universo
se inician en $t = t_1$ y
terminan en $t = t_2$.



- 1) Σ — ~~la~~ superficie de dos dimensiones del espacio físico, equivalente topológicamente a una esfera.
- 2) R — Volumen tridimensional del espacio físico encerrado por Σ .
- 3) Σ, R fijos para el observador O .
- 4) Γ — hipersuperficie definida por Σ y las líneas de universo de los puntos de Σ desde el tiempo t_0 hasta el tiempo t .
 Todos los cortes $t = \text{const.}$ de Γ son la ~~hipersuperficie~~ superficie Σ .

5) ④ ——— región ~~cuadr~~bidimensional
~~de~~ formada por las
 líneas de universo de los
 puntos de la región R
 entre el tiempo t_0 y
 el tiempo t .



$$\square h_{ij} = 8\pi$$

$$\square h_{ij} = 4\pi$$

$$\square h_{ij} = 4\pi M [2M]$$

$$\square h_{ij} = 4\pi M [2V_i V_j]$$

$$\square h_{ij} = 4\pi g [2V_i V_j - \Delta_{ij}]$$

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$$\Delta^{mn} h_{mn} = \frac{M[2-4]}{(\vec{V} \cdot \vec{r})} = \frac{-2M}{(\vec{V} \cdot \vec{r})}$$

$$\square \Delta^{mn} h_{mn} = -8\pi\mathcal{S}$$

$$-4\pi\mathcal{S} \Delta_{ij} = \frac{1}{2} \square \Delta^{mn} h_{mn} \Delta_{ij}$$

$$\square \left[h_{ij} - \frac{1}{2} \Delta^{mn} h_{mn} \Delta_{ij} \right] = 8\pi\mathcal{S} V_i V_j$$

$$\iiint_{\oplus} \square \left[h_{ij} - \frac{1}{2} \Delta^{mn} h_{mn} \Delta_{ij} \right] d\tau =$$

$$\iiint_{\oplus} 8\pi\mathcal{S} V_i V_j d\tau$$

6) $d\bar{v}$ — — — ~~elemento~~ elemento de hipervolumen (cuadridimensional)

7) dv — — — elemento de volumen tridimensional de la región R

8) $d\tau = dv dt$

$$\int \int \int \int \square \left[h_{ij} - \frac{1}{2} \Delta^{mn} h_{mn} \Delta_{ij} \right] d\tau$$

(H)

$$= \int \int \int \int 8\pi \rho V_i V_j dv dt$$

(H)

$$\Delta^{mn} h_{mn} = h$$

$$\int \int \int \int \square \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] d\tau =$$

(H)

$$\int \int \int \int 8\pi \rho V_i V_j dv dt$$

(H)

g) $d\Theta$ — incremento de la
 región cuatridimensional
 ~~$d\Theta$~~ cuando el tiempo
 va de t a $t + dt$.

$$d\Theta = R dt.$$

$$\iiint_{d\Theta} \square [h_{ij} - \frac{1}{2} h \Delta_{ij}] d\tau =$$

$$\iiint_{d\Theta} 8\pi \rho V_i V_j dv dt =$$

$$\int_R dt \int_R 8\pi \rho V_i V_j dv$$

$$\iiint_{d\Theta} \square \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] d\tau =$$

$$\iiint_{d\Gamma} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma}$$

10) $d\Gamma$ ——— hipersuperficie que encierra a $d\Theta$.

$d\Gamma$ está constituida por el corte de Θ para el tiempo t que es la región R ; el corte ~~del tiempo~~ de Θ para el tiempo $t+dt$ que también es la región R y por la banda lateral formada por los arcos de líneas de universo de los puntos de Σ entre los tiempos t y $t+dt$.

$$\iiint_{d\Theta} \square \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] d\tau = \iiint_{d\Theta} 8\pi G V_i V_j d\tau dt \quad 31$$

$$= \iiint_{d\Gamma} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma} =$$

$$\iiint_{R(t+dt)} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma}$$

$$- \iiint_{R(t)} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma}$$

$$+ \iiint_{\text{banda}} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma}$$

$$\text{banda} = \sum dt$$

$$\left| \iiint_{\text{banda}} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma} \right| \leq \sum dt \left| \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \right|$$

$$\lim_{dt \rightarrow 0} \iiint_{\text{banda}} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma} = 0$$

$$dt \iiint_R 8\pi \rho V_i V_j d\mathbf{v}$$

ii) $\hat{\mathbf{v}}$ ————— vector unitario a lo largo del ~~de~~ eje de los tiempos

$$d\vec{\Gamma} = \cancel{d\mathbf{v}} d\mathbf{v} \hat{\mathbf{v}} dt$$

$$dt \iiint_R 8\pi \rho V_i V_j d\mathbf{v} =$$

$$dt \iiint_R \left[\frac{d}{dt} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot \hat{\mathbf{v}} \right] d\mathbf{v}$$

$$8\pi g \vec{V}_i \vec{V}_j = \left\{ \frac{\partial}{\partial t} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \right\} \cdot \hat{n} \quad (33)$$

~~8\pi g~~

$$\frac{\partial}{\partial t} [8\pi g \vec{V}_i \vec{V}_j] = \left[\frac{\partial^2}{\partial t^2} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \right] \cdot \hat{n}$$

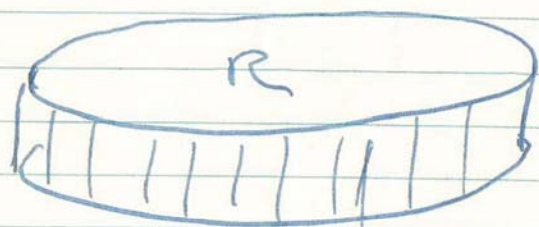
~~$$8\pi g (\vec{V}_i \vec{A}_j + \vec{A}_i \vec{V}_j) \cdot \hat{n}$$~~

$$8\pi g (\vec{V}_i \vec{A}_j + \vec{A}_i \vec{V}_j) \frac{dS}{dt} + 8\pi \frac{\partial \mathcal{L}}{\partial t} \vec{V}_i \vec{V}_j$$

$$= \frac{\partial^3}{\partial t^3} \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right]$$

$$\frac{\partial}{\partial t} [8\pi g \vec{V}_i \vec{V}_j]$$

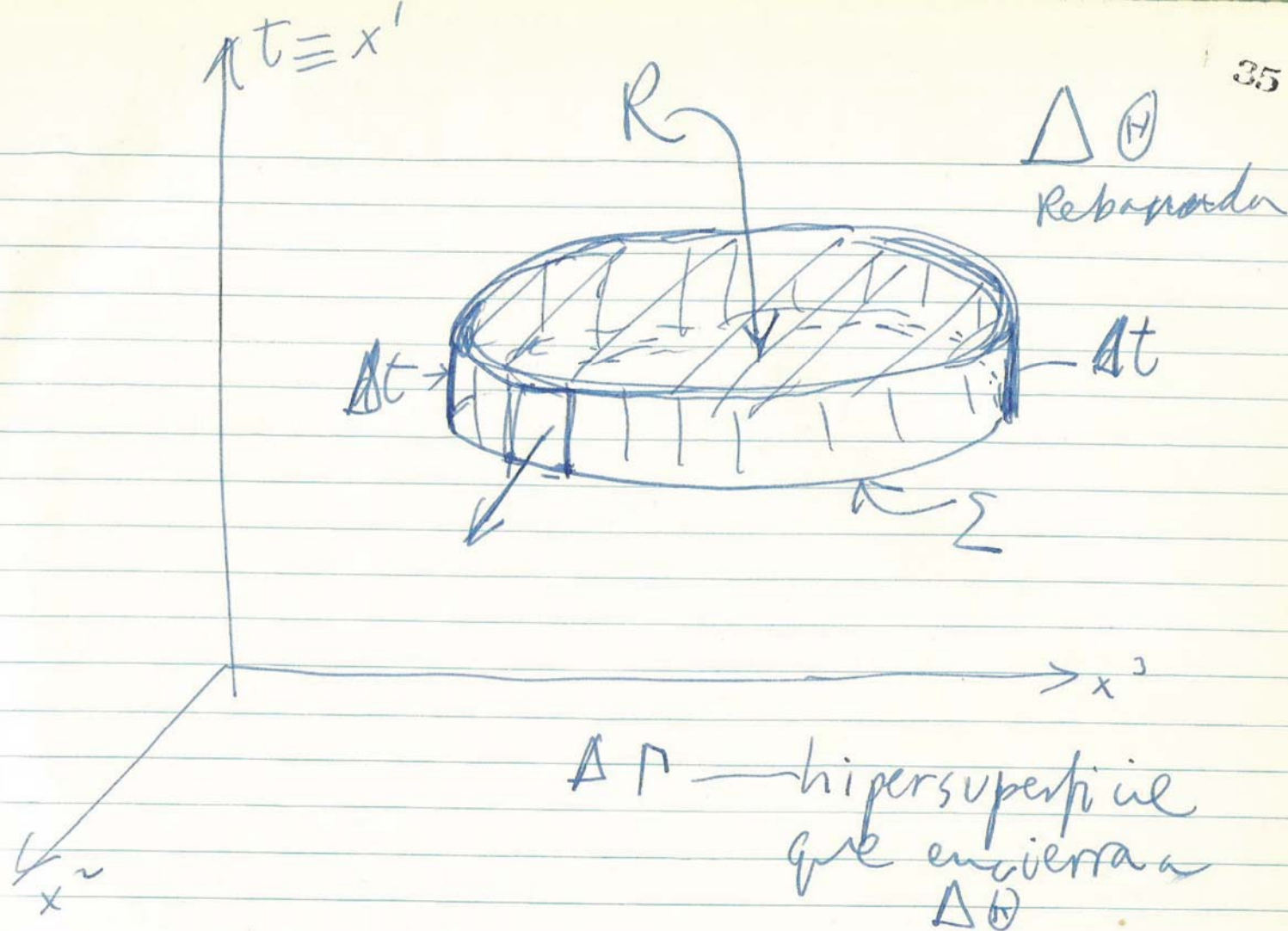
$$\iiint \square \left\{ h_{ij} - \frac{1}{2} h \Delta_{ij} \right\} d\tau =$$



$$\iiint 8\pi \rho \vec{V}_i \vec{V}_j d\tau = dt \iiint_R \frac{\partial}{\partial t} \nabla \left\{ h_{ij} - \frac{1}{2} h \Delta_{ij} \right\} d\vec{v}$$

$$\iint dt \iiint_R 8\pi \rho \vec{V}_i \vec{V}_j d\vec{v} = \iint dt \left[\frac{\partial}{\partial t} \nabla \left\{ h_{ij} - \frac{1}{2} h \Delta_{ij} \right\} \right] d\vec{v}$$

$\iint$



$$\iiint_{\Delta(H)} \square \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] d\tau = \iiint_{\Delta\Gamma} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma}$$

$$\begin{aligned} & \iiint_{\Delta(H)} \left[\frac{\partial^2}{\partial t^2} \left\{ h_{ij} - \frac{1}{2} h \Delta_{ij} \right\} - \Delta \left\{ h_{ij} - \frac{1}{2} h \Delta_{ij} \right\} \right] d\tau \\ &= \iiint_{\Delta\Gamma} \nabla \left[h_{ij} - \frac{1}{2} h \Delta_{ij} \right] \cdot d\vec{\Gamma} \end{aligned}$$

$$\iiint_{\Delta \Theta} \frac{\partial^2}{\partial t^2} H d\tau - \iiint_{\Delta \Theta} \Delta \{H\} d\tau$$

$$= \Delta t \iiint_R \frac{\partial^2}{\partial t^2} H dv + \iiint_{\text{Banda}} \vec{\nabla} H \cdot d\vec{\Gamma}$$

$$\cancel{\Delta t \iiint_R \frac{\partial^2}{\partial t^2} H d\tau} - \cancel{\iiint_{\Delta \Theta} \Delta H}$$

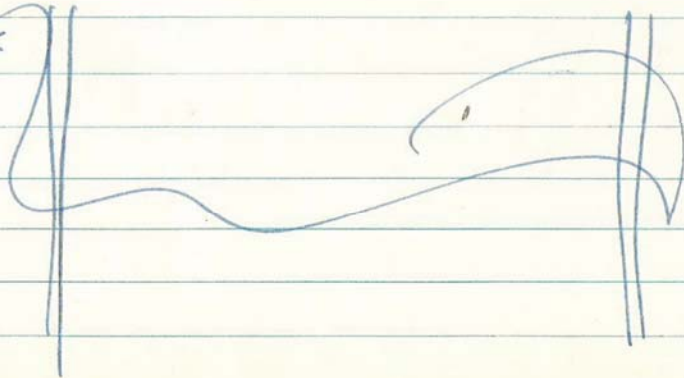
$$- \Delta t \iiint_R \Delta H dv = \cancel{\Delta t \iiint_R \frac{\partial^2}{\partial t^2} H dv}$$

$$+ \iiint_{\text{Banda}} \vec{\nabla} H \cdot d\vec{\Gamma}$$

$$\cancel{d\vec{\Gamma} = \vec{e}_i dx^i}$$

$$d\vec{\Gamma}_i = \vec{e}_{jkl} dx^j dx^k dx^l$$

$$d\vec{\Gamma} =$$



$$\begin{vmatrix} \Delta t & 0 & 0 & 0 \\ 0 & dx^2 & dx^3 & dx^4 \\ 0 & \delta x^2 & \delta x^3 & \delta x^4 \end{vmatrix}$$

$$d\vec{\Gamma} = \left(0, -\frac{dx^2 dx^4}{\delta x^2 \delta x^4} \Delta t, \frac{dx^2 dx^3}{\delta x^2 \delta x^3} \Delta t, -\frac{dx^3 dx^4}{\delta x^3 \delta x^4} \Delta t \right)$$

$$d\vec{\Gamma} =$$

$$d\vec{\Gamma} = (0, -d\vec{\Sigma} \Delta t)$$

componentes covariantes.

$$-\Delta t \iiint_R \Delta H dv = -\Delta t \int_{\Sigma} \vec{\nabla} H \cdot d\vec{\Sigma}$$

$$\iiint_R \Delta H dv = \int_{\Sigma} \vec{\nabla} H \cdot d\vec{\Sigma}$$

$$\Delta H = 2$$

$$\square H = \frac{\partial^2 H_{ij}}{\partial t^2} - \Delta H_{ij} = 8\pi g \bar{V}_i \bar{V}_j$$

$$\iiint_R \frac{\partial H}{\partial t^2} \left[\frac{\partial^2 H_{ij}}{\partial t^2} - \square H_{ij} \right] dV = \iint_{\Sigma} \vec{\nabla} H_{ij} \cdot d\vec{\Sigma}$$

$$\iiint_R \left[\frac{\partial^2 H_{ij}}{\partial t^2} - 8\pi g \bar{V}_i \bar{V}_j \right] dV = \iint_{\Sigma} \vec{\nabla} H_{ij} \cdot d\vec{\Sigma}$$

$$\iiint_R \left[\frac{\partial^2 H}{\partial t^2} - 8\pi g \frac{\partial}{\partial t} 8\pi g \bar{V}_i \bar{V}_j \right] dV$$

$$= \iint_{\Sigma} \vec{\nabla} \frac{\partial H}{\partial t} \cdot d\vec{\Sigma}$$

$$H = 8\pi g \bar{V}_i \bar{V}_j$$

$$\frac{\partial}{\partial t} H = 8\pi \frac{\partial g}{\partial t} \bar{V}_i \bar{V}_j + 8\pi g \bar{V}_i \frac{\partial \bar{V}_j}{\partial t} + 8\pi g \bar{V}_j \frac{\partial \bar{V}_i}{\partial t}$$

Calcular $\frac{\partial^3 H}{\partial t^3}$

$$H = h_{ij} - \frac{1}{2} h \Delta_{ij}$$

$$H_{ij} = h_{ij} - \frac{1}{2} \Delta^{mn} h_{mn} \Delta_{ij}$$

$$\frac{\partial^3 H_{ij}}{\partial t^3} = \frac{\partial^3 h_{ij}}{\partial t^3} - \frac{1}{2} \Delta^{mn} \frac{\partial^3 h_{mn}}{\partial t^3} \Delta_{ij}$$

$$\frac{\partial}{\partial t} F = \hat{\nabla} F \cdot \hat{j} \leftarrow \begin{array}{l} \text{vector unitario} \\ \text{al eje} \end{array}$$

$$\frac{\partial^2}{\partial t^2} F = \hat{\nabla} \{ \hat{\nabla} F \cdot \hat{j} \} \cdot \hat{j} \leftarrow \begin{array}{l} \text{a lo largo} \\ \text{del eje de} \\ \text{los tiempos.} \end{array}$$

$$\frac{\partial^2 F}{\partial t^2} \sim \hat{\nabla} \{ \hat{W} \cdot \hat{U} \} = ?$$

$$\hat{\nabla} \{ \hat{U} \cdot \hat{W} \} = ?$$

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$$\vec{\nabla}(\vec{U} \cdot \vec{W}) = \vec{\nabla}_U(\vec{U} \cdot \vec{W}) + \vec{\nabla}_W(\vec{U} \cdot \vec{W})$$

$$\vec{\nabla}_U(\vec{U} \cdot \vec{W}) = \vec{\nabla}_U \times (\vec{U} \times \vec{W})$$

$$(\vec{\nabla}_U \times \vec{U}) \times \vec{W} = (\vec{W} \cdot \vec{\nabla}_U) \vec{U} - \nabla$$

$$(\vec{\nabla}_U \times \vec{U}) \times \vec{W} = \vec{\nabla}_U(\vec{U} \cdot \vec{W}) - (\vec{W} \cdot \vec{\nabla}_U) \vec{U} - \nabla_U(\vec{U} \cdot \vec{W})$$

$$(\vec{\nabla}_W \times \vec{W}) \times \vec{U} = (\vec{U} \cdot \vec{\nabla}_W) \vec{W} - \nabla_W(\vec{U} \cdot \vec{W})$$

$$\nabla(\vec{U} \cdot \vec{W}) = \left\{ \begin{aligned} &(\vec{W} \cdot \nabla) \vec{U} + (\vec{U} \cdot \nabla) \vec{W} \\ &+ \vec{U} \times \text{rot } \vec{W} + \vec{W} \times \text{rot } \vec{U} \end{aligned} \right.$$

$$\vec{\nabla} F = \left(\frac{\partial F}{\partial t}, \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)$$

$$\vec{v} = (v^1, v^2, v^3, v^4)$$

$$\vec{\nabla} F \cdot \vec{v} = v^1 \frac{\partial F}{\partial t} + v^2 \frac{\partial F}{\partial x} + v^3 \frac{\partial F}{\partial y} + v^4 \frac{\partial F}{\partial z}$$

$$\frac{\partial}{\partial t} (\vec{\nabla} F \cdot \vec{v})$$

$$\frac{\partial}{\partial t} (\vec{\nabla} F \cdot \vec{v}) = v^1 \frac{\partial^2 F}{\partial t^2} + v^2 \frac{\partial^2 F}{\partial x \partial t} + v^3 \frac{\partial^2 F}{\partial y \partial t} + v^4 \frac{\partial^2 F}{\partial z \partial t}$$

$$\vec{v} = \left(\frac{x}{r^n}, \frac{y}{r^n}, \frac{z}{r^n} \right)$$

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$$\operatorname{div} \vec{v} = \left[\frac{1}{r^n} - \frac{n x^2}{r^{n+2}} + \frac{1}{r^n} - \frac{n y^2}{r^{n+2}} + \frac{1}{r^n} - \frac{n z^2}{r^{n+2}} \right]$$

$$\operatorname{div} \vec{v} = \frac{3r^2 - nr^2}{r^{n+2}} = \frac{(3-n)}{r^n}$$

$$\boxed{n=1}$$

$$\operatorname{div} \vec{v} = \frac{2}{r}$$

$$\vec{v} = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right)$$

$$\operatorname{div} \vec{v} = \left(\frac{1}{r} - \frac{x^2}{r^3} + \frac{1}{r} - \frac{y^2}{r^3} + \frac{1}{r} - \frac{z^2}{r^3} \right)$$

$$\operatorname{div} \vec{v} = \frac{3-1}{r} = \frac{2}{r}$$

$$\vec{v} = \left(\frac{x}{2r}, \frac{y}{2r}, \frac{z}{2r} \right)$$

$$\operatorname{div} \vec{v} = \frac{1}{r}$$

$$\nabla \cdot \vec{v} = \nabla \cdot \left(0, \frac{x}{2r}, \frac{y}{2r}, \frac{z}{2r} \right) = \frac{1}{r}$$

$$\hat{u} = \frac{\hat{r} - (\hat{r} \cdot \hat{v}) \hat{v}}{(\hat{r} \cdot \hat{v})}$$

$$\hat{u} = \frac{\hat{r}}{(\hat{r} \cdot \hat{v})} - \hat{v}$$

$$\text{div } \hat{u} = ?$$

$$\nabla \cdot (\hat{r} \cdot \hat{v}) =$$

$$1) h_{jk} = \frac{M [2 \vec{V}_j \vec{V}_k - \Delta_{jk}]}{(\vec{V} \cdot \vec{r})}$$

$$2) h = \Delta^{jk} h_{jk} = - \frac{2M}{(\vec{V} \cdot \vec{r})}$$

$$3) H_{jk} = h_{jk} - \frac{1}{2} h \Delta_{jk} = \frac{2M \vec{V}_j \vec{V}_k}{(\vec{V} \cdot \vec{r})}$$

$$4) H_{11} = h_{11} - \frac{1}{2} h = \frac{2M V_1 V_1}{(\vec{V} \cdot \vec{r})}$$

$$5) \text{ Calculer } \frac{\partial^2}{\partial t^2} H_{11}$$

$$6) \iiint_R \left[\frac{\partial^2}{\partial t^2} H_{11} - \frac{8}{\pi} \rho V_1 V_1 \right] dV = \iint_\Sigma \vec{\nabla} H_{11} \cdot d\vec{\Sigma}$$

$$H_{11} = \frac{2M \vec{V} \cdot \vec{r}}{(\vec{V} \cdot \vec{r})}$$

$$\vec{V} = (V^1, V^2, V^3, V^4)$$

$$\vec{V} = \left(\frac{1}{\sqrt{1-V^2}}, \frac{V^1}{\sqrt{1-V^2}}, \frac{V^2}{\sqrt{1-V^2}}, \frac{V^3}{\sqrt{1-V^2}} \right)$$

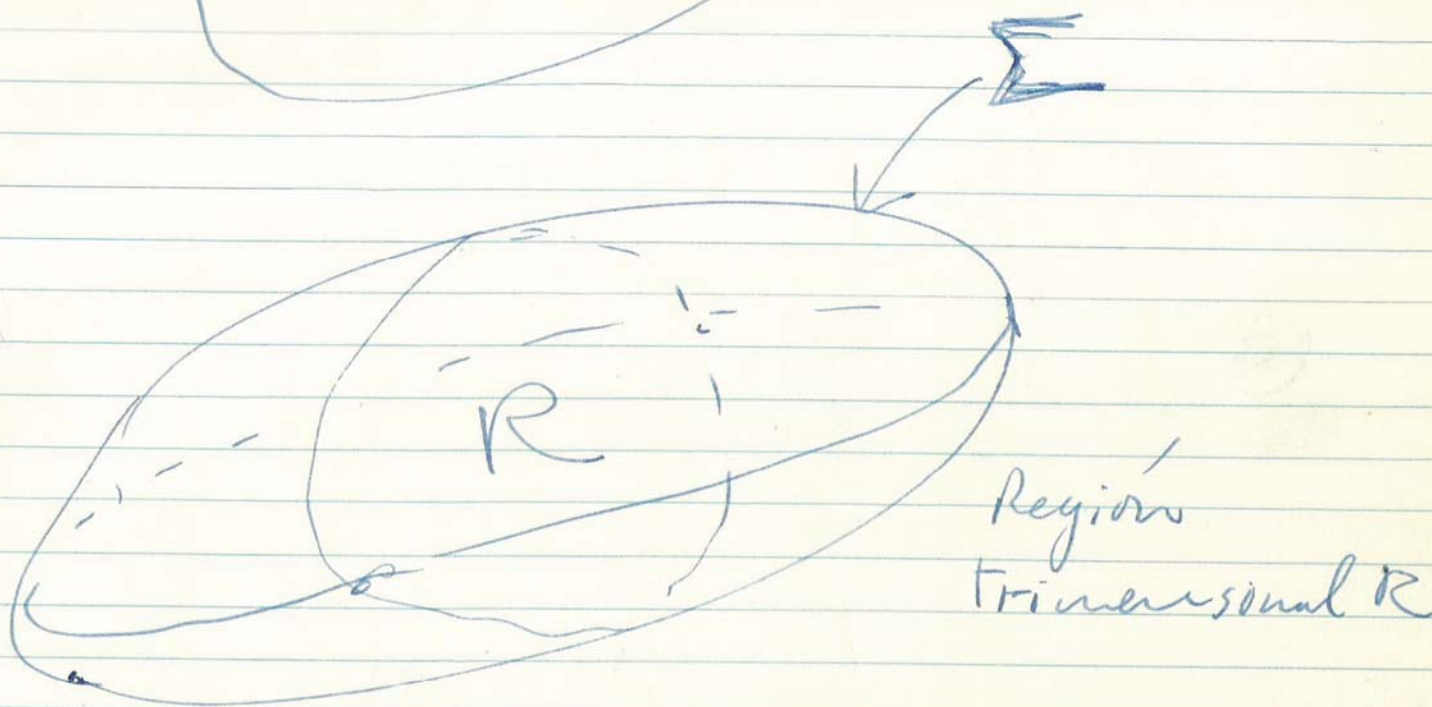
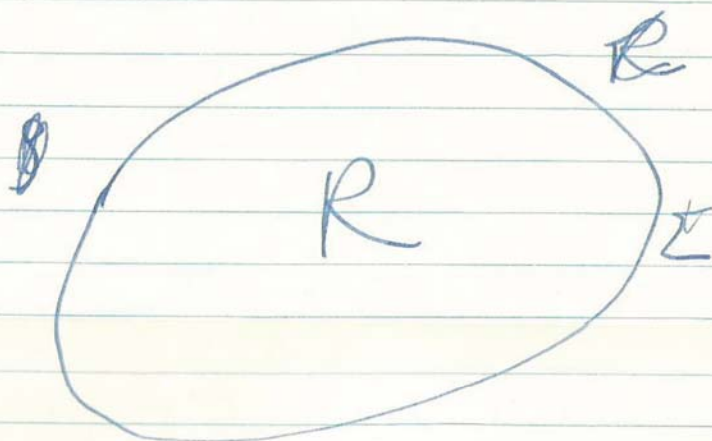
$$\vec{r} = (t-T, \underline{x-X}, y-Y, z-Z)$$

$$(\vec{V} \cdot \vec{r}) = \frac{r - \vec{V} \cdot \vec{r}}{\sqrt{1-V^2}}$$

$$H_{11} = \frac{2M(r - \vec{V} \cdot \vec{r})}{\sqrt{1-V^2}}$$

$$H_{11} = \frac{2M}{[r - (\vec{V} \cdot \vec{r})] \sqrt{1-V^2}}$$

$$\iiint_R \frac{2M}{[r - (\vec{v} \cdot \vec{r})] \sqrt{1 - v^2}} dv = ?$$



$$\iiint_R \frac{\rho_0 V_1 V_2}{\sqrt{1 - v^2}} dv = ?$$

$$\iiint_R \rho_0 \bar{V}_1 \bar{V}_1 d\mathbf{v} = ?$$

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$$\iiint \frac{\rho_0}{1 - V^2} d\mathbf{v}$$

$$8\pi \rho_0 \bar{V}_1 \bar{V}_1 = \square H_{11}$$

$$\frac{1}{8\pi} \iiint_R \square H_{11} d\mathbf{v}$$

$$\frac{1}{8\pi} \iiint_R \frac{\partial^2}{\partial t^2} H_{11} d\mathbf{v} - \frac{1}{8\pi} \iiint_R \Delta H_{11} d\mathbf{v}$$

$$\iiint_R \frac{\partial^2}{\partial t^2} H_{11} d\mathbf{v}$$

$$\iiint \vec{\nabla} H \cdot d\vec{\Gamma}$$

Banda

$$\vec{r} \neq$$

$$\boxed{\frac{\partial}{\partial t} = \hat{r} \cdot \vec{\nabla}}$$

$$\frac{\partial}{\partial t} H = (\hat{r} \cdot \vec{\nabla}) H = \hat{r} \cdot \vec{\nabla} H$$

$$\frac{\partial^2}{\partial t^2} H = \{\hat{r} \cdot \vec{\nabla}\} \{\hat{r} \cdot \vec{\nabla} H\}$$

$$\left(\frac{\partial H}{\partial t}, \frac{\partial H}{\partial x}, \frac{\partial H}{\partial y}, \frac{\partial H}{\partial z} \right)$$

$$v^1 \frac{\partial H}{\partial t} + v^2 \frac{\partial H}{\partial x} + v^3 \frac{\partial H}{\partial y} + v^4 \frac{\partial H}{\partial z}$$

$$\hat{\gamma} \cdot \hat{\nabla} \{ \hat{\gamma} \cdot \hat{\nabla} H \} = \gamma^m \gamma^n \frac{\partial^2 H}{\partial x^m \partial x^n}$$

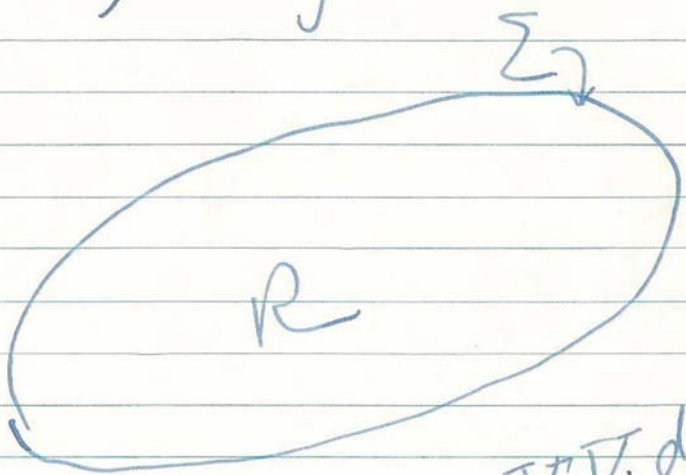
$$\gamma' \gamma' \frac{\partial^2 H}{\partial t \partial t}$$

$$+ 2\gamma' \gamma^x \frac{\partial^2 H}{\partial t \partial x}$$

$$\hat{\gamma} \cdot \hat{\nabla} \{ \hat{\gamma} \cdot \hat{\nabla} H \} = \gamma^m \gamma^n \frac{\partial^2 H}{\partial x^m \partial x^n}$$

$$\hat{\nabla} \{ \hat{\nabla} \cdot \hat{\nabla} H \} = \left[\begin{aligned} & \nu^1 \frac{\partial^2 H}{\partial t^2} + \nu^2 \frac{\partial^2 H}{\partial t \partial x} + \nu^3 \frac{\partial^2 H}{\partial t \partial y} \\ & \quad + \nu^4 \frac{\partial^2 H}{\partial t \partial z}, \\ & \nu^1 \frac{\partial^2 H}{\partial t \partial x} + \nu^2 \frac{\partial^2 H}{\partial x^2} + \nu^3 \frac{\partial^2 H}{\partial x \partial y} \\ & \quad + \nu^4 \frac{\partial^2 H}{\partial x \partial z}, \\ & \nu^1 \frac{\partial^2 H}{\partial t \partial y} + \nu^2 \frac{\partial^2 H}{\partial x \partial y} + \nu^3 \frac{\partial^2 H}{\partial y^2} \\ & \quad + \nu^4 \frac{\partial^2 H}{\partial y \partial z}, \\ & \nu^1 \frac{\partial^2 H}{\partial t \partial z} + \nu^2 \frac{\partial^2 H}{\partial x \partial z} + \nu^3 \frac{\partial^2 H}{\partial y \partial z} \\ & \quad + \nu^4 \frac{\partial^2 H}{\partial z^2} \end{aligned} \right]$$

$$\oint_0 \vec{V}_i \cdot \vec{V}_j$$

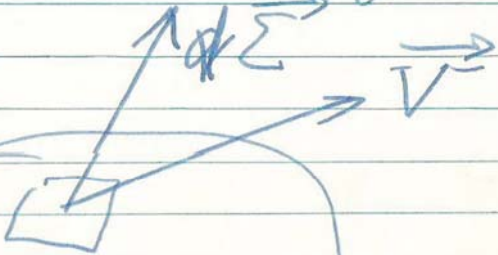


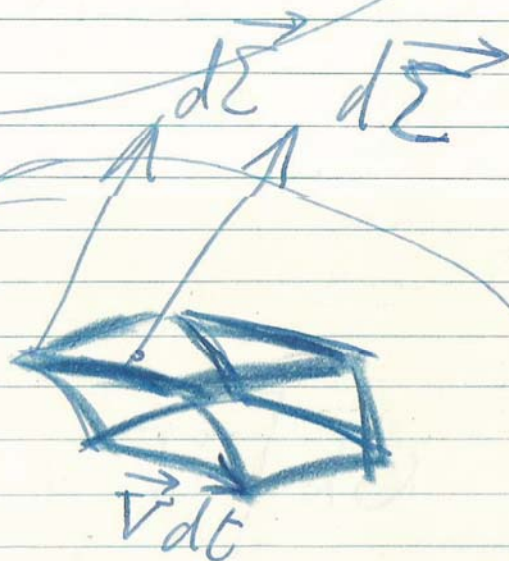
$$\oint_0 \vec{V}_i \cdot \vec{V}_j \vec{V}$$

$$\oint_0 \vec{V}_i \cdot \vec{V}_j d\vec{v}$$



$$d\Sigma \vec{v}$$





$$\vec{v} dt \cdot d\vec{\Sigma} \rightarrow dv$$

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$$\rho_0 V_i V_j \vec{v} \cdot d\vec{\Sigma} dt$$

$$F_{ij} = \iint_{\Sigma} \rho_0 V_i V_j \vec{v} \cdot d\vec{\Sigma}$$

rapidez
con la
que sale
energía momento
de la
región R.

$$E_{ij} = \iint_R \frac{\partial}{\partial t} [\rho_0 V_i V_j] dv$$

rapidez
con la
que aumenta
la energía
momento
en R

$$E_{ij} = \iint_R \frac{\partial}{\partial t} [\rho_0 V_i V_j] dv$$

$$\frac{\partial}{\partial t} [\epsilon_0 \vec{V}_i \vec{V}_j] = \left\{ \frac{\partial \epsilon_0}{\partial t} \vec{V}_i \vec{V}_j + \epsilon_0 A_i \vec{V}_j \frac{\vec{r}}{(\vec{r} \cdot \vec{r})} \right. \\ \left. + \epsilon_0 \vec{V}_i A_j \frac{\vec{r}}{(\vec{r} \cdot \vec{r})} \right\}$$

$$\iiint_R \left(\frac{\partial \epsilon_0}{\partial t} \vec{V}_i \vec{V}_j + \epsilon_0 (A_i \vec{V}_j + \vec{V}_i A_j) \frac{\vec{r}}{(\vec{r} \cdot \vec{r})} \right) d\tau$$



$$\iiint_{d\oplus} \epsilon_0 \vec{V}_i \vec{V}_j d\tau =$$

~~$$\iiint_R \frac{\partial}{\partial t} (\epsilon_0 \vec{V}_i \vec{V}_j) d\tau + \iiint_R \epsilon_0 (\vec{V}_i \vec{A}_j + \vec{A}_i \vec{V}_j) \frac{\vec{r}}{(\vec{r} \cdot \vec{r})} d\tau$$~~

$$\iiint \rho_0 \vec{V}_i \vec{V}_j \sim$$

$$\vec{\nabla} \{ \rho_0 \vec{V}_i \vec{V}_j \} \cdot d\vec{\Sigma}$$

$$\iiint_{\Sigma} \vec{\nabla} \{ \rho_0 \vec{V}_i \vec{V}_j \} \cdot d\vec{\Sigma} =$$

$$\frac{d}{dt} \iiint_R \frac{\partial}{\partial t} \{ \rho_0 \vec{V}_i \vec{V}_j \} d\vec{v}$$

$$+ \iiint_{\text{Banda}} \vec{\nabla} \{ \rho_0 \vec{V}_i \vec{V}_j \} \cdot d\vec{\Sigma}$$

$$\iiint_{\Sigma} \vec{\nabla} \{ \rho_0 \vec{V}_i \vec{V}_j \} \cdot d\vec{\Sigma} =$$

$$\frac{d}{dt} \iiint_R E_{ij} dt$$

$$\iiint \hat{\nabla} \{s_0 \bar{V}_i \bar{V}_j\} \cdot d\hat{\Sigma} =$$

~~Bound~~

$$E_{ij} dt + \iiint_{\text{Bound}} \hat{\nabla} \{s_0 \bar{V}_i \bar{V}_j\} \cdot d\hat{\Sigma}$$

$$d\hat{\Sigma}_i = [0, -d\vec{\Sigma} dt] \text{ Comp. covariant}$$

$$d\hat{\Sigma}_i = [0, -d\vec{\Sigma} dt]$$

$$\hat{\nabla} \{s_0 \bar{V}_i \bar{V}_j\} = \left[\frac{\partial}{\partial t} (s_0 \bar{V}_i \bar{V}_j), -\vec{\nabla} (s_0 \bar{V}_i \bar{V}_j) \right]$$

$$\hat{\nabla} \{s_0 \bar{V}_i \bar{V}_j\} \cdot d\hat{\Sigma} = \vec{\nabla} (s_0 \bar{V}_i \bar{V}_j) \cdot d\vec{\Sigma} dt$$

$$\iiint_{\Sigma} \hat{\nabla} \{s_0 \bar{V}_i \bar{V}_j\} \cdot d\hat{\Sigma} =$$

$$E_{ij} dt + dt \iint_{\Sigma} \vec{\nabla} (s_0 \bar{V}_i \bar{V}_j) \cdot d\vec{\Sigma}$$

$$\iiint \hat{\nabla} \{s_0 \vec{V}_i \vec{V}_j\} \cdot d\vec{\Sigma} =$$

$$E_{ij} dt + dt \iint_{\Sigma} \vec{\nabla} (s_0 \vec{V}_i \vec{V}_j) \cdot d\vec{\Sigma}$$

$$\iiint_{\Pi} \hat{\nabla} \{s_0 \vec{V}_i \vec{V}_j\} \cdot d\vec{\Sigma} =$$

$$\iiint_{d \oplus} \square \{s_0 \vec{V}_i \vec{V}_j\} d\tau$$

$$= \# \iiint_R \square \square H_{ij} dv$$

$$E_{ij} dt =$$

$$E_{ij} = \iiint_R \square \square H_{ij} dv - \iint_{\Sigma} \vec{\nabla} (\underline{\underline{H_{ij}}}) \cdot d\vec{\Sigma}$$

En el espacio vacío.

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$$\iiint_{\mathbb{R}} \square H_{ij} d\tau = 0$$

$$\iiint_{\mathbb{R}} \frac{\partial^2}{\partial t^2} H_{ij} d\tau = + \iiint_{\mathbb{R}} \Delta \square H_{ij} d\tau \quad \boxed{24}$$

$$\iiint_{\mathbb{R}} \frac{\partial^2}{\partial t^2} H_{ij} d\tau + 8\pi \iiint_{\mathbb{R}} \square \rho_0 V_i V_j d\tau$$

$$\iiint_{\mathbb{R}} \frac{\partial^2}{\partial t^2} H_{ij} d\tau = \iiint_{\mathbb{R}} \Delta \left(\int H_{ij} dt \right) d\mathbb{R}$$

$$8\pi \iiint \frac{\partial}{\partial t} \rho_0 V_i V_j d\tau = \iiint \Delta$$

$$\iiint_{\mathbb{R}} \frac{\partial^2}{\partial t^2} \rho_0 V_i V_j$$

$$\iiint_{\mathbb{R}} \frac{\partial^2}{\partial t^2} \rho_0 V_i V_j d\tau = \iiint_{\mathbb{R}} \Delta \rho_0 V_i V_j d\tau$$

$$+ \iiint_{\mathbb{R}} \square \rho_0 V_i V_j d\tau$$

$\vec{F}_i$ — fuerza total aplicada
a la partícula i

$$\vec{F}_i = 0$$

$$\vec{F}_i \cdot \delta \vec{r}_i = 0$$

$$\vec{F}_i = \vec{F}_i^{(a)} + \vec{f}_i \quad \text{— restricción}$$

$$0 = \vec{F}_i \cdot \delta \vec{r}_i = \vec{F}_i^{(a)} \cdot \delta \vec{r}_i + \vec{f}_i \cdot \delta \vec{r}_i$$

$$\vec{F}_i^{(a)} \cdot \delta \vec{r}_i = 0$$

$\vec{f}_i \cdot \delta \vec{r}_i$ — trabajo virtual de
las fuerzas de
restricción.

El trabajo virtual total de
las fuerzas aplicadas es nulo

$$\sum \vec{F}_i^{(a)} \neq 0$$

$$\vec{F}_i = \dot{\vec{p}}_i$$

$$\vec{F}_i - \dot{\vec{p}}_i = 0$$

fuerza
total

en la partícula i

reversed effective force

$$(\vec{F}_i - \dot{\vec{p}}_i) \cdot \delta \vec{r}_i = 0$$

$$\vec{F}_i = \vec{F}_i^{(a)} + \vec{f}_i$$

$$(\vec{F}_i^{(a)} + \vec{f}_i - \dot{\vec{p}}_i) \cdot \delta \vec{r}_i = 0$$

$$(\vec{F}_i^{(a)} - \dot{\vec{p}}_i) \cdot \delta \vec{r}_i + (\vec{f}_i \cdot \delta \vec{r}_i) = 0$$

$$(\vec{F}_i^{(a)} - \dot{\vec{p}}_i) \cdot \delta \vec{r}_i = 0$$

$$\vec{r}_i = (q_1, q_2, \dots, q_n, t)$$

$$\vec{v}_i = \frac{\partial \vec{r}_i}{\partial q_j} \dot{q}_j + \frac{\partial \vec{r}_i}{\partial t}$$

$$\delta \vec{r}_i = \frac{\partial \vec{r}_i}{\partial q_j} \delta q_j$$

$$\vec{F}_i \cdot \delta \vec{r}_i = \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} \delta q_j$$

$$\vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} = Q_j$$

$$\vec{r}_i = \vec{r}_i(q_1, q_2, q_3, \dots, q_n, t)$$

$$\vec{v}_i = \frac{\partial \vec{r}_i}{\partial q_j} \dot{q}_j + \frac{\partial \vec{r}_i}{\partial t}$$

$$\boxed{\frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \frac{\partial \vec{r}_i}{\partial q_j}}$$

$$\frac{\partial \vec{r}_i}{\partial q_j} = \frac{\partial \left(\frac{d}{dt} \vec{r}_i \right)}{\partial \left(\frac{d}{dt} q_j \right)} = \frac{\partial \vec{v}_i}{\partial \dot{q}_j}$$

$$\boxed{\frac{d}{dt} \frac{\partial \vec{r}_i}{\partial \dot{q}_j}}$$

$$\frac{\partial \vec{v}_i}{\partial q_k} = \frac{\partial^2 \vec{r}_i}{\partial q_j \partial q_k} \dot{q}_j + \frac{\partial^2 \vec{r}_i}{\partial q_k \partial t}$$

$$\frac{d}{dt} \frac{\partial \vec{r}_i}{\partial q_k} = \frac{\partial^2 \vec{r}_i}{\partial q_j \partial q_k} \ddot{q}_j + \frac{\partial^2 \vec{r}_i}{\partial q_k \partial t}$$

$$\left[\frac{\partial \vec{r}_i}{\partial \dot{q}_j} \right]$$

$$\vec{v}_i = \frac{\partial \vec{r}_i}{\partial q_j} \dot{q}_j + \frac{\partial \vec{r}_i}{\partial t}$$

$$\left[\frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \frac{\partial \vec{r}_i}{\partial q_j} \right]$$

$$\frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \frac{\partial \frac{d}{dt}(\vec{r}_i)}{\partial \frac{d}{dt}(q_j)} = \frac{\partial \vec{r}_i}{\partial q_j}$$

$$\frac{d}{dt} \frac{\partial \vec{r}_i}{\partial q_j} = \frac{\partial^2 \vec{r}_i}{\partial q_j \partial q_k} \dot{q}_k + \frac{\partial^2 \vec{r}_i}{\partial q_j \partial t}$$

$$\frac{\partial \vec{v}_i}{\partial \dot{q}_k} = \frac{\partial^2 \vec{r}_i}{\partial q_j \partial q_k} \dot{q}_j + \frac{\partial^2 \vec{r}_i}{\partial q_k \partial t}$$

$$\left[\frac{d}{dt} \frac{\partial \vec{r}_i}{\partial q_j} = \frac{\partial \vec{v}_i}{\partial \dot{q}_j} \right]$$

$$\sum (\vec{F}_i^{(a)} - \dot{\vec{p}}_i) \cdot \delta \vec{r}_i = 0$$

$$\# \sum \vec{F}_i^{(a)} \cdot \delta \vec{r}_i$$

$$\sum \vec{F}_i^{(a)} \cdot \frac{\partial \vec{r}_i}{\partial q_j} \delta q_j$$

$$Q_j = \sum_i \vec{F}_i^{(a)} \cdot \frac{\partial \vec{r}_i}{\partial q_j}$$

$$(Q_j \delta q_j)$$

$$\vec{p}_i = m_i \vec{v}_i = m_i \dot{\vec{r}}_i$$

$$\vec{p}_i \cdot \delta \vec{r}_i = m_i \dot{\vec{r}}_i \cdot \delta \vec{r}_i$$

$$m_i \vec{v}_i \cdot \delta \vec{r}_i$$

$$\delta m_i \vec{v}_i \cdot \delta \vec{r}_i =$$

$$\sum m_i \vec{v}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} \delta q_j$$

$$\sum m_i \vec{v}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} = \sum \left[\frac{d}{dt} \left(m_i \vec{v}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j} \right) - m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial q_j} \right]$$

$$\sum m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \left[\frac{d}{dt} \left(m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j} \right) - m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j} \right] \quad 62$$

$$\sum m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \frac{d}{dt} (\cancel{T})$$

$$\frac{d}{dt} \frac{\partial}{\partial \dot{q}_j} (\cancel{T}) = m_i \vec{v}_i \cdot \frac{\partial \vec{v}_i}{\partial \dot{q}_j}$$

$$+ \frac{\partial h_{ij}}{\partial x^k}$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^3} 110 - \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 110$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^2} 120 + \frac{2M}{(\vec{V} \cdot \vec{r})^2} 210$$

$$- \frac{M}{(\vec{V} \cdot \vec{r})^3} 450 + \frac{M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 450$$

$$- \frac{2M}{(\vec{V} \cdot \vec{r})^2} 111 + \frac{M}{(\vec{V} \cdot \vec{r})^2} 451$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^3} 011$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^2} 012$$

$$+ \frac{M}{(\vec{V} \cdot \vec{r})^3} 013$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^2} 111$$

$$a_i = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) \frac{dx^j}{ds} \frac{dx^k}{ds}$$

$$h_{jk} = \frac{M[2\vec{V}_j \cdot \vec{V}_k - \Delta_{jk}]}{(\vec{V} \cdot \vec{r})}$$

$$h_{ij} = \frac{M[2\vec{V}_i \cdot \vec{V}_j - \Delta_{ij}]}{(\vec{V} \cdot \vec{r})}$$

$$- \frac{\partial h_{jk}}{\partial x^i}$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^3} 011 + \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 011$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^2} 012 - \frac{2M}{(\vec{V} \cdot \vec{r})^2} 021$$

$$+ \frac{M}{(\vec{V} \cdot \vec{r})^3} 045 - \frac{M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 045$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^2} 111 - \frac{M}{(\vec{V} \cdot \vec{r})^2} 145$$

$$\left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

$$- \frac{2M}{(\vec{V} \cdot \vec{r})^3} 011 + \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 011 - \frac{2M}{(\vec{V} \cdot \vec{r})^2} 012$$

$$- \frac{2M}{(\vec{V} \cdot \vec{r})^2} 021 + \frac{M}{(\vec{V} \cdot \vec{r})^3} 045 - \frac{M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 045$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{r})^3} 110 - \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 110 + \frac{2M}{(\vec{V} \cdot \vec{r})^2} 120$$

$$- \frac{M}{(\vec{V} \cdot \vec{r})^2} 145 + \frac{2M}{(\vec{V} \cdot \vec{r})^2} 210 - \frac{M}{(\vec{V} \cdot \vec{r})^3} 450$$

$$+ \frac{M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} 450 + \frac{M}{(\vec{V} \cdot \vec{r})^2} 451$$

$$\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}$$

$$\begin{aligned} & - \frac{2M}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \vec{V}_j \vec{V}_k + \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \vec{V}_j \vec{V}_k - \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{r}_i \vec{V}_j \vec{A}_k \\ & - \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{r}_i \vec{A}_j \vec{V}_k + \frac{M}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \Delta_{jk} - \frac{M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \Delta_{jk} \\ & + \frac{2M}{(\vec{V} \cdot \vec{A})^3} \vec{V}_i \vec{V}_j \vec{r}_k - \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \vec{V}_i \vec{V}_j \vec{r}_k + \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \vec{A}_j \vec{r}_k \\ & - \frac{M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \Delta_{jk} + \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{A}_i \vec{V}_j \vec{r}_k - \frac{M}{(\vec{V} \cdot \vec{A})^3} \Delta_{ij} \vec{r}_k \\ & + \frac{M(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \Delta_{ij} \vec{r}_k + \frac{M}{(\vec{V} \cdot \vec{A})^2} \Delta_{jk} \vec{V}_i \end{aligned}$$

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$$h^{jk} = \frac{M[2\vec{V}^j \vec{V}^k - \Delta^{jk}]}{(\vec{V} \cdot \vec{A})}$$

$$h^{jk} = \frac{2M\vec{V}^j \vec{V}^k}{(\vec{V} \cdot \vec{A})} - \frac{M\Delta^{jk}}{(\vec{V} \cdot \vec{A})}$$

$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

$$-\frac{4M^2}{(\vec{V} \cdot \vec{r})^4} \cdot \vec{r}_i + \frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i - \frac{4M^2}{(\vec{V} \cdot \vec{r})^3} \vec{r}_i + \frac{2M^2}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i$$

$$-\frac{2M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i + \frac{4M^2}{(\vec{V} \cdot \vec{r})^2} \vec{A}_i - \frac{2M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i + \frac{2M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i$$

$$-\frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i + \frac{4M^2}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i - \frac{4M^2}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i$$

$$-\frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i + \frac{4M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i - \frac{2M^2}{(\vec{V} \cdot \vec{r})^2} \vec{A}_i$$

$$-\frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i + \frac{4M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i - \frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i$$

$$+\frac{2M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i + \frac{2M^2}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i - \frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i$$

$$+\frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i - \frac{2M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i + \frac{2M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i$$

$$+\frac{M^2}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i - \frac{M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \vec{r}_i - \frac{4M^2}{(\vec{V} \cdot \vec{r})^3} \vec{V}_i$$

$$\left[h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] \right]$$

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Términos en

$$\begin{aligned} & - \frac{4M^2}{(\vec{V} \cdot \vec{A})^4} + \frac{4M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^4} + \frac{2M^2}{(\vec{V} \cdot \vec{A})^4} - \frac{2M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^4} \\ & + \frac{2M^2}{(\vec{V} \cdot \vec{A})^4} - \frac{2M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^4} - \frac{4M^2}{(\vec{V} \cdot \vec{A})^4} + \frac{4M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^4} \\ & + \frac{M^2}{(\vec{V} \cdot \vec{A})^4} - \frac{M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^4} \end{aligned}$$

$$\left[- \frac{3M^2}{(\vec{V} \cdot \vec{A})^4} + \frac{3M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^4} \right] \vec{r}_i$$

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$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

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Terminos en V_i

$$+ \frac{4M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$- \frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3}$$

$$- \frac{2M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$+ \frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3}$$

$$- \frac{2M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$+ \frac{2M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$- \frac{2M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$+ \frac{2M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$+ \frac{4M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$- \frac{4M^2}{(\vec{V} \cdot \vec{r})^3}$$

$$\left[+ \frac{2M^2}{(\vec{V} \cdot \vec{r})^3} - \frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} \right] V_i$$

• Equivocado!

$$\left[h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] \right]$$

Términos en A_i

$$+ \frac{2M^2}{(V_{1r})^2} A_i$$

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Resultado:

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$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

[1]

$$+ \left[-\frac{3M^2}{(\vec{V} \cdot \vec{A})^4} + \frac{3M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^4} \right] \vec{r}_i$$

[2]

$$+ \left[\frac{2M^2}{(\vec{V} \cdot \vec{A})^3} - \frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \right] \vec{V}_i$$

[3]

$$+ \left[\frac{2M^2}{(\vec{V} \cdot \vec{A})^2} \right] A_i$$

Programa

1) Comprobar resultado

2) Calcular $\square h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$

$$= \square \left[[1] r_i + [2] V_i + [3] A_i \right]$$

$$- h^{jk} \square \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] - \nabla h^{jk} \cdot \left[\frac{\partial}{\partial x^k} \nabla h_{ij} - \frac{\partial}{\partial x^i} \nabla h_{jk} \right]$$

$$h^{jk} \frac{\partial h_{ij}}{\partial x^k} - h^{jk} \frac{\partial h_{jk}}{\partial x^i}$$

$$\frac{\partial}{\partial x^k} (h_{ij} h^{jk}) = h^{jk} \frac{\partial h_{ij}}{\partial x^k} + h_{ij} \frac{\partial h^{jk}}{\partial x^k}$$

$$\frac{\partial}{\partial x^i} (h^{jk} h_{jk}) = h^{jk} \frac{\partial h_{jk}}{\partial x^i} + h_{jk} \frac{\partial h^{jk}}{\partial x^i}$$

$$-\frac{\partial h_{jk}}{\partial x_i}$$

$$-\frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{r}_i \vec{V}_j A_k - \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{r}_i A_j \vec{V}_k$$

$$+ \frac{2M(\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \vec{V}_j \vec{V}_k - \frac{M(\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \Delta_{jk}$$

$$= -\frac{2M}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \vec{V}_j \vec{V}_k + \frac{M}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \Delta_{jk}$$

$$+ \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \vec{V}_j \vec{V}_k - \frac{M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \Delta_{jk}$$

$$\frac{\partial h_{jk}}{\partial x^i}$$

$$\begin{aligned} & \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{r}_i \vec{V}_j A_k^V + \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{r}_i A_j \vec{V}_k + \\ & - \frac{2M(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \vec{V}_j \vec{V}_k + \frac{M(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \Delta_{jk} \\ & + \frac{2M}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \vec{V}_j \vec{V}_k - \frac{M}{(\vec{V} \cdot \vec{A})^3} \vec{r}_i \Delta_{jk} \\ & - \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \vec{V}_j \vec{V}_k + \frac{M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \Delta_{jk} \end{aligned}$$

$$\frac{\partial h_{ij}}{\partial x^k}$$

$$\begin{aligned} & + \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i A_j \vec{r}_k + \frac{2M}{(\vec{V} \cdot \vec{A})^2} A_i \vec{V}_j \vec{r}_k \\ & - \frac{2M(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \vec{V}_i \vec{V}_j \vec{r}_k + \frac{M(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \Delta_{ij} \vec{r}_k \\ & + \frac{2M}{(\vec{V} \cdot \vec{A})^3} \vec{V}_i \vec{V}_j \vec{r}_k - \frac{M}{(\vec{V} \cdot \vec{A})^3} \Delta_{ij} \vec{r}_k \\ & - \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \vec{V}_j \vec{V}_k + \frac{M}{(\vec{V} \cdot \vec{A})^2} \Delta_{ij} \vec{V}_k \end{aligned}$$

$$\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i}$$

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$$\begin{aligned} & + \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \vec{A}_j r_k + \frac{2M}{(\vec{V} \cdot \vec{A})^2} \vec{A}_i \vec{V}_j r_k - \frac{2M}{(\vec{V} \cdot \vec{A})^2} r_i \vec{V}_j \vec{A}_k - \frac{2M}{(\vec{V} \cdot \vec{A})^2} r_i \vec{A}_j \vec{V}_k \\ & - \frac{2M(\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} \vec{V}_i \vec{V}_j r_k + \frac{M(\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} \Delta_{ij} r_k + \frac{2M(\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} r_i \vec{V}_j \vec{V}_k - \frac{M(\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} r_i \Delta_{jk} \\ & + \frac{2M}{(\vec{V} \cdot \vec{A})^3} \vec{V}_i \vec{V}_j r_k - \frac{M}{(\vec{V} \cdot \vec{A})^3} \Delta_{ij} r_k - \frac{2M}{(\vec{V} \cdot \vec{A})^3} r_i \vec{V}_j \vec{V}_k + \frac{M}{(\vec{V} \cdot \vec{A})^3} r_i \Delta_{jk} \\ & + \frac{M}{(\vec{V} \cdot \vec{A})^2} \Delta_{ij} \vec{V}_k - \frac{M}{(\vec{V} \cdot \vec{A})^2} \vec{V}_i \Delta_{jk} \end{aligned}$$

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$$h^{jk} = \frac{M[2\vec{V}^j \vec{V}^k - \Delta^{jk}]}{(\vec{V} \cdot \vec{A})}$$

$$h^{jk} = \frac{2M\vec{V}^j \vec{V}^k}{(\vec{V} \cdot \vec{A})} - \frac{M\Delta^{jk}}{(\vec{V} \cdot \vec{A})}$$

$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

Términos en r_i

$$\begin{aligned}
 & - \frac{4M^2}{(\vec{V} \cdot \vec{r})^4} & + \frac{4M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \\
 & + \frac{2M^2}{(\vec{V} \cdot \vec{r})^4} & - \frac{2M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \\
 & + \frac{M^2}{(\vec{V} \cdot \vec{r})^4} & - \frac{M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \\
 & + \frac{2M^2}{(\vec{V} \cdot \vec{r})^4} & - \frac{2M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \\
 & - \frac{4M^2}{(\vec{V} \cdot \vec{r})^4} & + \frac{4M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4}
 \end{aligned}$$

$$\left[- \frac{3M^2}{(\vec{V} \cdot \vec{r})^4} + \frac{3M^2(\hat{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \right] r_i$$

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$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

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Términos en V_i

$$+ \frac{4M^2}{(\vec{V} \cdot \vec{A})^3}$$

$$- \frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3}$$

$$- \frac{2M^2}{(\vec{V} \cdot \vec{A})^3}$$

$$+ \frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3}$$

$$- \frac{2M^2}{(\vec{V} \cdot \vec{A})^3}$$

$$- \frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3}$$

$$- \frac{M^2}{(\vec{V} \cdot \vec{A})^3}$$

$$+ \frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3}$$

$$+ \frac{4M^2}{(\vec{V} \cdot \vec{A})^3}$$

$$\left[\frac{3M^2}{(\vec{V} \cdot \vec{A})^3} - \frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{A})^3} \right] V_i$$

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$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

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Términos en A_i

$$+ \frac{2M^2}{(V_{,i}A)^2} A_i$$

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Segunda Comprobación

$$\left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

$$\begin{aligned}
 & - \frac{2M}{(\vec{r} \cdot \vec{r})^3} \vec{r}_i \vec{r}_j \vec{r}_k + \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{r} \cdot \vec{r})^3} \vec{r}_i \vec{r}_j \vec{r}_k - \frac{2M}{(\vec{r} \cdot \vec{r})^2} \vec{r}_i \vec{r}_j \vec{A}_k - \frac{2M}{(\vec{r} \cdot \vec{r})^2} \vec{r}_i \vec{A}_j \vec{r}_k \\
 & + \frac{M}{(\vec{r} \cdot \vec{r})^3} \vec{r}_i \Delta_{jk} - \frac{M(\vec{A} \cdot \vec{r})}{(\vec{r} \cdot \vec{r})^3} \vec{r}_i \Delta_{jk} \\
 & + \frac{2M}{(\vec{r} \cdot \vec{r})^3} \vec{r}_i \vec{r}_j \vec{r}_k - \frac{2M(\vec{A} \cdot \vec{r})}{(\vec{r} \cdot \vec{r})^3} \vec{r}_i \vec{r}_j \vec{r}_k + \frac{2M}{(\vec{r} \cdot \vec{r})^2} \vec{r}_i \vec{A}_j \vec{r}_k - \frac{M}{(\vec{r} \cdot \vec{r})^2} \vec{r}_i \Delta_{jk} \\
 & + \frac{2M}{(\vec{r} \cdot \vec{r})^2} \vec{A}_i \vec{r}_j \vec{r}_k - \frac{M}{(\vec{r} \cdot \vec{r})^3} \Delta_{ij} \vec{r}_k + \frac{M(\vec{A} \cdot \vec{r})}{(\vec{r} \cdot \vec{r})^3} \Delta_{ij} \vec{r}_k \\
 & + \frac{M}{(\vec{r} \cdot \vec{r})^2} \Delta_{ij} \vec{r}_k
 \end{aligned}$$

$$\boxed{h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j} \right]}$$

$$- \frac{4M^2}{(V \cdot A)^4} r_i + \frac{4M^2(A \cdot A)}{(V \cdot A)^4} r_i - \frac{4M^3}{(V \cdot A)^3} r_i - \frac{4M^2 \cdot 0}{(V \cdot A)^3} r_i$$

$$+ \frac{2M^2}{(V \cdot A)^4} r_i - \frac{2M^2(A \cdot A)}{(V \cdot A)^4} r_i$$

$$+ \frac{4M^2}{(V \cdot A)^3} V_i - \frac{4M^2(A \cdot A)}{(V \cdot A)^3} V_i + \frac{4M^2 \cdot 0}{(V \cdot A)^2} V_i - \frac{2M^2}{(V \cdot A)^3} V_i$$

$$+ \frac{4M^2}{(V \cdot A)^2} A_i - \frac{2M^2}{(V \cdot A)^3} V_i + \frac{2M^2(A \cdot A)}{(V \cdot A)^3} V_i$$

$$+ \frac{2M^2}{(V \cdot A)^2} V_i + \frac{2M}{(V \cdot A)^4} r_i - \frac{2M^2(A \cdot A)}{(V \cdot A)^4} r_i + \frac{2M^2 \cdot 0}{(V \cdot A)^3} r_i$$

$$- \frac{4M^2}{(V \cdot A)^4} r_i + \frac{4M^2(A \cdot A)}{(V \cdot A)^4} r_i - \frac{2M^2}{(V \cdot A)^3} V_i + \frac{2M^2(A \cdot A)}{(V \cdot A)^3} V_i - \frac{2M^2(A \cdot A)}{(V \cdot A)^3} V_i$$

$$+ \frac{4M^2}{(V \cdot A)^3} V_i - \frac{2M^2}{(V \cdot A)^2} A_i + \frac{M^2}{(V \cdot A)^4} r_i - \frac{M^2}{(V \cdot A)^3} r_i$$

$$h^{ik} \left[\frac{\partial h^{ij}}{\partial x^k} - \frac{\partial h^{ik}}{\partial x^j} \right]$$

Términos en V_i

$$+ \frac{4M^2}{(V \cdot \hat{A})^3} - \frac{4M^2(A \cdot \hat{A})}{(V \cdot \hat{A})^3}$$

$$- \frac{2M^2}{(V \cdot \hat{A})^3} + \frac{2M^2(A \cdot \hat{A})}{(V \cdot \hat{A})^3}$$

$$+ \frac{2M^2}{(V \cdot \hat{A})^3} - \frac{2M^2(A \cdot \hat{A})}{(V \cdot \hat{A})^3}$$

$$+ \frac{4M^2}{(V \cdot \hat{A})^3} - \frac{M^2}{(V \cdot \hat{A})^3}$$

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$$\left[\frac{3M^2}{(V \cdot A)^3} - \frac{2M^2(A \cdot A)}{(V \cdot A)^2} \right] V_i$$

Resultado

$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

$$+ \left[- \frac{3M^2}{(\vec{V} \cdot \vec{r})^4} + \frac{3M^2 (\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4} \right] \vec{r}_i$$

$$+ \left[+ \frac{3M^2}{(\vec{V} \cdot \vec{r})^3} - \frac{2M^2 (\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^3} \right] \vec{V}_i$$

$$+ \left[\frac{2M^2}{(\vec{V} \cdot \vec{r})^2} \right] A_i$$

$$\frac{dS}{dx^j}$$

87

$$\hat{r} = (x^1 - X^1, x^2 - X^2, x^3 - X^3, x^4 - X^4)$$

$$\hat{r} \cdot \hat{r} = 0$$

$$\hat{r} = \hat{s} - \hat{R}$$

$$r^i = x^i - X^i$$

$$\frac{\partial r^i}{\partial x^j} = \delta_j^i - V^i \frac{\partial S}{\partial x^j}$$

~~$$\Delta_{mn} r^m r^n = \hat{r} \cdot \hat{r} = 0$$~~

~~$$\frac{\partial}{\partial x^j} [\Delta_{mn} r^m r^n] = \Delta_{mn} \left[\delta_j^m - V^m \frac{\partial S}{\partial x^j} \right] r^n + \Delta_{mn} r^m \left[\delta_j^n - V^n \frac{\partial S}{\partial x^j} \right]$$~~

$$\frac{\partial}{\partial x^j} [\Delta_{mn} r^m r^n] = \left(\Delta_{mn} \left(\delta_j^m - V^m \frac{\partial S}{\partial x^j} \right) r^n + \Delta_{mn} r^m \left(\delta_j^n - V^n \frac{\partial S}{\partial x^j} \right) \right)$$

$$0 = \cancel{r_j} - (\hat{V} \cdot \hat{r}) \frac{\partial S}{\partial x_j} + r_j - (\hat{V} \cdot \hat{r}) \frac{\partial S}{\partial x_j}$$

$$\boxed{\frac{\partial S}{\partial x_j} = \frac{r_j}{(\hat{V} \cdot \hat{r})}}$$

Comprobado
cuaderno (F2)
pág. 8.

$$\frac{\partial E}{\partial x_j} = \Delta_{ij} - \frac{1}{(\hat{V} \cdot \hat{r})} \cancel{V_i} r_j$$

$$\boxed{\frac{\partial E}{\partial x_j} = \Delta_{ij} - \frac{1}{(\hat{V} \cdot \hat{r})} V_i r_j}$$

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(F2) pág. 9.

$$\frac{\partial V_i}{\partial x_j} = A_i \frac{r_j}{(\hat{V} \cdot \hat{r})} = \frac{1}{(\hat{V} \cdot \hat{r})} A_i r_j$$

$$\boxed{\frac{\partial V_i}{\partial x_j} = \frac{1}{(\hat{V} \cdot \hat{r})} A_i r_j}$$

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(F2) pág. 8

$$\boxed{\frac{\partial A_i}{\partial x_j} = \frac{1}{(\hat{V} \cdot \hat{r})} B_i r_j}$$

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(F2) pág. 9.

$$\delta_{ij}, r_{ij}, V_{ij}, A_{ij}$$

82

$$\hat{V} \cdot \hat{r}, (\hat{A} \cdot \hat{r}), (\hat{V} \cdot \hat{r})^{-1}$$

$$\frac{\partial}{\partial x_j} [\Delta^{mn} V_m r_n] = \Delta^{mn} \left[\frac{1}{(\hat{V} \cdot \hat{r})} A_m r_j r_n + V_m \left\{ \Delta_{nj} - \frac{1}{(\hat{V} \cdot \hat{r})} V_n r_j \right\} \right]$$

$$\frac{\partial}{\partial x_j} [\hat{V} \cdot \hat{r}] = \frac{(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})} r_j + V_j - \frac{1}{(\hat{V} \cdot \hat{r})} r_j$$

$$\frac{\partial}{\partial x_j} [\hat{V} \cdot \hat{r}] = -\frac{1}{(\hat{V} \cdot \hat{r})} r_j + \frac{(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})} r_j + V_j$$

Comprovações cuadermo
(F2) pág. 10

$$\frac{\partial}{\partial x_j} [\Delta^{mn} A_m r_n] = \Delta^{mn} \left[\frac{1}{(\hat{V} \cdot \hat{r})} B_m r_j r_n + A_m \left\{ \Delta_{nj} - \frac{1}{(\hat{V} \cdot \hat{r})} V_n r_j \right\} \right]$$

$$\frac{\partial}{\partial x_j} (\hat{A} \cdot \hat{r}) = \frac{(\hat{B} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})} \hat{r}_j + A_j$$

Cuaderno

83

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$$\frac{\partial}{\partial x_j} \left[\frac{1}{(\hat{V} \cdot \hat{r})^2} \right] = -\frac{2}{(\hat{V} \cdot \hat{r})^3} \left[-\frac{1}{(\hat{V} \cdot \hat{r})} \hat{r}_j + \frac{(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})} \hat{r}_j + V_j \right]$$

$$\frac{\partial}{\partial x_j} \left[\frac{1}{(\hat{V} \cdot \hat{r})^2} \right] = +\frac{2}{(\hat{V} \cdot \hat{r})^4} \hat{r}_j - \frac{2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4} \hat{r}_j - \frac{2}{(\hat{V} \cdot \hat{r})^3} V_j$$

$$\frac{\partial}{\partial x_j} \left[\frac{1}{(\hat{V} \cdot \hat{r})^3} \right] = -\frac{3}{(\hat{V} \cdot \hat{r})^4} \left[-\frac{1}{(\hat{V} \cdot \hat{r})} \hat{r}_j + \frac{(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})} \hat{r}_j + V_j \right]$$

$$\frac{\partial}{\partial x_j} \left[\frac{1}{(\hat{V} \cdot \hat{r})^3} \right] = +\frac{3}{(\hat{V} \cdot \hat{r})^5} \hat{r}_j - \frac{3(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5} \hat{r}_j - \frac{3}{(\hat{V} \cdot \hat{r})^4} V_j$$

$$\frac{\partial}{\partial x_j} \frac{1}{(\hat{V} \cdot \hat{r})^4} = -\frac{4}{(\hat{V} \cdot \hat{r})^5} \left[-\frac{1}{(\hat{V} \cdot \hat{r})} \hat{r}_j - \frac{(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})} \hat{r}_j + V_j \right]$$

$$\frac{\partial}{\partial x_j} \left[\frac{1}{(\hat{V} \cdot \hat{r})^4} \right] = +\frac{4}{(\hat{V} \cdot \hat{r})^6} \hat{r}_j + \frac{4(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^6} \hat{r}_j - \frac{4}{(\hat{V} \cdot \hat{r})^5} V_j$$

Cuadrivector $L^{\hat{1}}$

Definición.

$$L_i = h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

$$L_i = \left[-\frac{3M^2}{(\vec{V} \cdot \vec{A})^4} \vec{r}_i + \frac{3M^2 (\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^4} \vec{r}_i + \frac{3M^2}{(\vec{V} \cdot \vec{A})^3} V_i - \frac{2M^2 (\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} V_i + \frac{2M^2}{(\vec{V} \cdot \vec{A})^2} A_i \right]$$

$$\frac{\partial}{\partial x_j} \left[- \frac{3M^2}{(\vec{V} \cdot \vec{A})^4} r_i \right]$$

$$\frac{\partial}{\partial x_j} \left[+ \frac{3M^2}{(\vec{V} \cdot \vec{A})^4} r_i \right]$$

$$\frac{\partial}{\partial x_j} \left[+ \frac{3M^2}{(\vec{V} \cdot \vec{A})^3} V_i \right]$$

$$\frac{\partial}{\partial x_j} \left[- \frac{2M^2 (\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})^3} V_i \right]$$

$$\frac{\partial}{\partial x_j} \left[+ \frac{2M^2}{(\vec{V} \cdot \vec{A})^2} A_i \right]$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $r_i r_j$

85

00

$$-\frac{12 M^2}{(\hat{V} \cdot \hat{r})^6} + \frac{12 M^2 (\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^6}$$

$$+ \frac{12 M^2 (\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^6} - \frac{12 M^2 (\hat{A} \cdot \hat{r})^2}{(\hat{V} \cdot \hat{r})^6} + \frac{3 M^2 (\hat{B} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5}$$

$\frac{\partial L_i}{\partial x_i}$ Términos en $r_i V_j$

86

01

$$+ \frac{12 M^2}{(V \cdot r)^5}$$

$$- \frac{12 M^2 (A \cdot F)}{(V \cdot r)^5}$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $\Gamma_i A_j$

87

02

$$+ \frac{3M^2}{(V \cdot r)^4}$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $V_i V_j$
10

88

$$+ \frac{3M^2}{(\hat{V} \cdot \hat{r})^5}$$

$$- \frac{3M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5}$$

$$+ \frac{9M^2}{(\hat{V} \cdot \hat{r})^5} - \frac{9M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5}$$

$$- \frac{6M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5} + \frac{6M^2(\hat{A} \cdot \hat{r})^2}{(\hat{V} \cdot \hat{r})^5} - \frac{2M^2(\hat{B} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4}$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $V_i V_j$

89

~~11~~

$$- \frac{9M^2}{(\hat{V} \cdot \hat{r})^4}$$

$$+ \frac{6M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4}$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $V_i A_j$

90

12

$$- \frac{2M^2}{(V \cdot A)^3}$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $A_i r_j$

91

20

$$+ \frac{3M^2}{(\vec{V} \cdot \vec{r})^4}$$

$$- \frac{2M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4}$$

$$+ \frac{4M^2}{(\vec{V} \cdot \vec{r})^4} - \frac{4M^2(\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4}$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $A_i V_j$

21

$$\frac{4M^2}{(V \cdot r)^3}$$

$\frac{\partial L_i}{\partial x_j}$ Términos en $B_i \Gamma_j$

93

30

$$+ \frac{2M^2}{(\vec{V} \cdot \vec{r})^3}$$

$\frac{\partial L_i}{\partial x_j}$ términos en A_{ij}

94

45

$$- \frac{3M^2}{(\vec{V} \cdot \vec{r})^4}$$

$$+ \frac{3M^2 (\vec{A} \cdot \vec{r})}{(\vec{V} \cdot \vec{r})^4}$$

$$\boxed{\frac{\partial L_i}{\partial x_j}}$$

Comp wación
pendiente.

95

$$+ \left[-\frac{12M^2}{(\hat{V} \cdot \hat{r})^6} \{1 - (\hat{A} \cdot \hat{r})\}^2 + \frac{3M^2(\hat{B} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5} \right] \hat{r}_i \hat{r}_j$$

$$+ \left[\frac{12M^2}{(\hat{V} \cdot \hat{r})^5} \{1 - (\hat{A} \cdot \hat{r})\} \right] \hat{r}_i \hat{V}_j$$

$$+ \left[\frac{3M^2}{(\hat{V} \cdot \hat{r})^4} \right] \hat{r}_i \hat{A}_j$$

$$+ \left[+\frac{12M^2}{(\hat{V} \cdot \hat{r})^5} - \frac{18M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5} + \frac{6M^2(\hat{A} \cdot \hat{r})^2}{(\hat{V} \cdot \hat{r})^5} - \frac{2M^2(\hat{B} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4} \right] \hat{r}_i \hat{r}_j$$

$$+ \left[-\frac{9M^2}{(\hat{V} \cdot \hat{r})^4} + \frac{6M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4} \right] \hat{V}_i \hat{V}_j$$

$$+ \left[-\frac{2M^2}{(\hat{V} \cdot \hat{r})^3} \right] \hat{V}_i \hat{A}_j$$

$$+ \left[+\frac{7M^2}{(\hat{V} \cdot \hat{r})^4} - \frac{6M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4} \right] \hat{A}_i \hat{r}_j$$

$$+ \left[-\frac{4M^2}{(\hat{V} \cdot \hat{r})^3} \right] \hat{A}_i \hat{V}_j + \left[\frac{2M^2}{(\hat{V} \cdot \hat{r})^3} \right] \hat{B}_i \hat{r}_j + \#$$

$$\boxed{\frac{\partial L_i}{\partial x_j}}$$

Compensación
pendiente.

95

$$+ \left[-\frac{12M^2}{(\hat{V} \cdot \hat{r})^6} \left\{ 1 - (\hat{A} \cdot \hat{r})^2 \right\} + \frac{3M^2(\hat{B} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5} \right] \hat{r}_i \hat{r}_j$$

$$+ \left[\frac{12M^2}{(\hat{V} \cdot \hat{r})^5} \left\{ 1 - (\hat{A} \cdot \hat{r})^2 \right\} \right] \hat{r}_i V_j$$

$$+ \left[\frac{3M^2}{(\hat{V} \cdot \hat{r})^4} \right] \hat{r}_i A_j$$

$$+ \left[+\frac{12M^2}{(\hat{V} \cdot \hat{r})^5} - \frac{18M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^5} + \frac{6M^2(\hat{A} \cdot \hat{r})^2}{(\hat{V} \cdot \hat{r})^5} - \frac{2M^2(\hat{B} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4} \right] \hat{r}_i \hat{r}_j$$

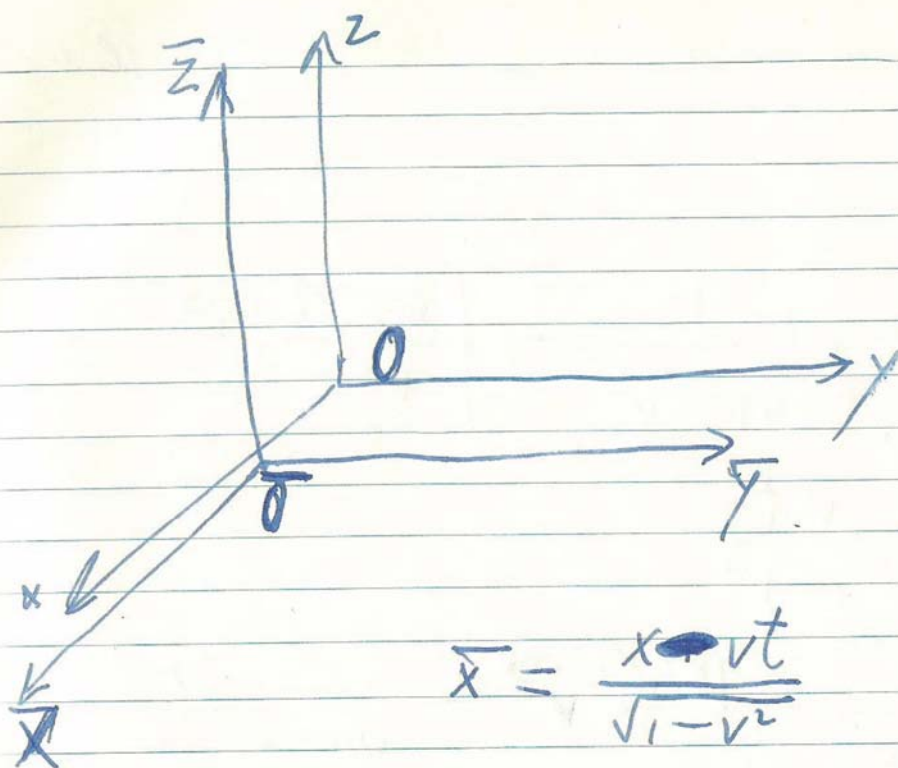
$$+ \left[-\frac{9M^2}{(\hat{V} \cdot \hat{r})^4} + \frac{6M^2(\hat{A} \cdot \hat{r})}{(\hat{V} \cdot \hat{r})^4} \right] V_i V_j$$

$$\Delta_{ij} \frac{\partial L_i}{\partial x_j}$$

$$\frac{3M^2 - 2M^2(\hat{A} \cdot \hat{r})}{(\hat{r} \cdot \hat{r})^4}$$

¡ojó!

Comprobación
pendiente



$$\bar{x} = \frac{x - vt}{\sqrt{1-v^2}} \quad \bar{t} = \frac{t - vx}{\sqrt{1-v^2}}$$

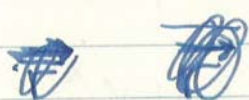
$$\bar{y} = y$$

$$\bar{z} = z$$

$$\bar{x} = x + \left\{ \frac{1}{\sqrt{1-v^2}} - 1 \right\} - \frac{v}{\sqrt{1-v^2}} t$$

$$\bar{y} = y$$

$$\bar{z} = z$$



$$x = \frac{\vec{r} \cdot \vec{v}}{v}$$

$\frac{\vec{v}}{v}$ --- vector unitario a lo largo del eje v_x

$$\frac{\vec{r} \cdot \vec{v}}{v} = x$$

$$\vec{r} = \left[\vec{r} + \left(\frac{1}{\sqrt{1-v^2}} - 1 \right) \frac{(\vec{r} \cdot \vec{v})}{v} \frac{\vec{v}}{v} - \frac{\vec{v} t}{\sqrt{1-v^2}} \right]$$

$$\vec{r} = \vec{r} + \left(\frac{1}{\sqrt{1-v^2}} - 1 \right) \frac{(\vec{r} \cdot \vec{v})}{v^2} \vec{v} - \frac{\vec{v}}{\sqrt{1-v^2}} t$$

$$t = \frac{1}{\sqrt{1-v^2}} t - \frac{(\vec{r} \cdot \vec{v})}{\sqrt{1-v^2}}$$

$$\bar{x} = x + \left[\frac{1}{\sqrt{1-v^2}} - 1 \right] \frac{xv^x + yv^y + zv^z}{v^2} v^x - \frac{v^x t}{\sqrt{1-v^2}}$$

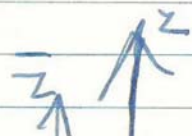
$$\bar{x} = -\frac{v^x}{\sqrt{1-v^2}} t + \left\{ 1 + \left(\frac{1}{\sqrt{1-v^2}} - 1 \right) \frac{1}{v^2} v^x v^x \right\} x + \left(\frac{1}{\sqrt{1-v^2}} - 1 \right) \frac{v^x v^y}{v^2} y + \left(\frac{1}{\sqrt{1-v^2}} - 1 \right) \frac{v^x v^z}{v^2} z$$

$$\frac{1}{\sqrt{1-v^2}}$$

$$\vec{r} = \vec{r} + \frac{\vec{r} \cdot \vec{v}}{v} \left\{ \frac{1}{\sqrt{1-v^2}} - 1 \right\} \frac{\vec{v}}{v} - \frac{v}{\sqrt{1-v^2}} \frac{\vec{v}}{v} t \quad 98$$

$$\vec{r} = \vec{r} + \frac{\vec{r} \cdot \vec{v}}{v} \left\{ \frac{1}{\sqrt{1-v^2}} - 1 \right\} \frac{\vec{v}}{v} - \frac{\vec{v}}{\sqrt{1-v^2}} t$$

$$\bar{t} = \frac{1}{\sqrt{1-v^2}} t - \frac{\vec{r} \cdot \vec{v}}{\sqrt{1-v^2}}$$



$$\bar{x} = \frac{x - vt}{\sqrt{1-v^2}};$$

$$\bar{y} = y; \quad \bar{z} = z.$$

$$\bar{t} = \frac{t - vx}{\sqrt{1-v^2}}$$

$$\bar{x} = x + \left[\frac{1}{\sqrt{1-v^2}} - 1 \right] x - \frac{v}{\sqrt{1-v^2}} t;$$

$$\bar{y} = y; \quad \bar{t} = \frac{t - vx}{\sqrt{1-v^2}}$$

$$\bar{z} = z;$$

$$\bar{t} = \beta t - \beta v^x x - \beta v^y y - \beta v^z z$$

$$\bar{x} = -\beta v^x t + \left[1 + \frac{\beta-1}{v^2} v^x v^x\right] x + \frac{\beta-1}{v^2} v^x v^y y + \frac{\beta-1}{v^2} v^x v^z z$$

$$\bar{y} = -\beta v^y t + \frac{\beta-1}{v^2} v^x v^y x + \left[1 + \frac{\beta-1}{v^2} v^y v^y\right] y + \frac{\beta-1}{v^2} v^y v^z z$$

$$\bar{z} = -\beta v^z t + \frac{\beta-1}{v^2} v^x v^z x + \frac{\beta-1}{v^2} v^y v^z y + \left[1 + \frac{\beta-1}{v^2} v^z v^z\right] z$$

$$\bar{t} = \frac{t}{\sqrt{1-v^2}} - \frac{\vec{v} \cdot \vec{r}}{\sqrt{1-v^2}} = \frac{t - \vec{v} \cdot \vec{r}}{\sqrt{1-v^2}}$$

$$\vec{r} = \vec{r} - \frac{\vec{v}}{\sqrt{1-v^2}} t + \frac{\beta-1}{v^2} (\vec{v} \cdot \vec{r}) \vec{v}$$

$$\bar{t} = \beta t - \beta V^x x - \beta V^y y - \beta V^z z ;$$

$$\bar{x} = \left[-\beta V^x t + \left[1 + \frac{\beta-1}{V^2} V^x V^x \right] x + \frac{\beta-1}{V^2} V^x V^y y + \frac{\beta-1}{V^2} V^x V^z z \right] ;$$

$$\bar{y} = \left[-\beta V^y t + \frac{\beta-1}{V^2} V^x V^y x + \left[1 + \frac{\beta-1}{V^2} V^y V^y \right] y + \frac{\beta-1}{V^2} V^y V^z z \right] ;$$

$$\bar{z} = \left[-\beta V^z t + \frac{\beta-1}{V^2} V^x V^z x + \frac{\beta-1}{V^2} V^y V^z y + \left[1 + \frac{\beta-1}{V^2} V^z V^z \right] z \right] ;$$

$$\beta = \frac{1}{\sqrt{1-V^2}} .$$

$$\bar{t} = \frac{t - \vec{V} \cdot \vec{r}}{\sqrt{1 - V^2}}$$

$$\vec{r} = \vec{r} - \frac{\vec{V}}{\sqrt{1 - V^2}} t + \frac{\beta - 1}{V^2} (\vec{V} \cdot \vec{r}) \vec{V}$$

$$\vec{v} = \frac{d\vec{r}}{d\bar{t}}$$

$$(1 + \vec{v} \cdot \vec{v}) [1 + (\vec{v})^2] d\bar{t}$$

$$d\bar{t} + d\vec{r} \cdot \frac{d\vec{r}}{d\bar{t}}$$

$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

$$\square h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] = h^{jk} \square \left[\right]$$

$$h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right]$$

Teorema de Green

$$\int_{\tau} \left(\square h^{jk} \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] - h^{jk} \square \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] \right) d\tau$$

$$= \int \left(\left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] \nabla h^{jk} - \left[\nabla \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) \right] h^{jk} \right) dA$$

$$\frac{\partial}{\partial x^k} V_i V_j - \frac{\partial}{\partial x^i} V_j V_k$$

$$\left(\frac{1}{(\vec{V} \cdot \vec{r})} A_i \cancel{r_j V_k} + \frac{1}{(\vec{V} \cdot \vec{r})} V_i \cancel{A_j r_k} - \frac{1}{(\vec{V} \cdot \vec{r})} A_j \cancel{r_i V_k} - \frac{1}{(\vec{V} \cdot \vec{r})} \cancel{V_j A_k} r_i \right) h^{jk}$$

$$\square \left[\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right] =$$

$$8\pi \frac{\partial}{\partial x^k} \left[V_i V_j - \frac{1}{2} \Delta_{ij} \right] - 8\pi \frac{\partial}{\partial x^i} \left[V_j V_k - \frac{1}{2} \Delta_{jk} \right]$$

$$\frac{F(s)}{(\vec{V} \cdot \vec{A})} = \Phi$$

$$\frac{\partial \Phi}{\partial x^i} = \frac{F'(s)}{(\vec{V} \cdot \vec{A}) (\vec{V} \cdot \vec{A})} r_i - \frac{F(s)}{(\vec{V} \cdot \vec{A})^2} \left[-\frac{r_i}{(\vec{V} \cdot \vec{A})} + \frac{(\vec{A} \cdot \vec{A})}{(\vec{V} \cdot \vec{A})} r_i + V_i \right]$$

$$\frac{\partial \Phi}{\partial x^i} = \left[\frac{F'(s)}{(\vec{V} \cdot \vec{A})^2} r_i + \frac{F(s)}{(\vec{V} \cdot \vec{A})^3} r_i - \frac{(\vec{A} \cdot \vec{A}) F(s)}{(\vec{V} \cdot \vec{A})^3} r_i \right. \\ \left. - \frac{F(s)}{(\vec{V} \cdot \vec{A})^2} V_i \right]$$

$$\Phi = \frac{F(s)}{(V \cdot A)^2} r_j + \frac{F(s)}{(V \cdot A)^3} r_j - \frac{F(s)}{(V \cdot A)^3} r_j$$

$$\frac{\partial \Phi}{\partial x_i \partial x_j} = \frac{F''(s)}{(V \cdot A)^3} r_j + \frac{2F'(s)}{(V \cdot A)^4} r_j - \frac{2F'(s)}{(V \cdot A)^4} r_j - \frac{2F'(s)}{(V \cdot A)^3} r_j$$

$$- \frac{F'(s)}{(V \cdot A)^3} r_j + \frac{F'(s)}{(V \cdot A)^2} \Delta_j$$

$$+ \frac{F'(s)}{(V \cdot A)^4} r_j + \frac{3F(s)}{(V \cdot A)^5} r_j - \frac{3F(s)}{(V \cdot A)^5} r_j - \frac{3F(s)}{(V \cdot A)^4} r_j$$

$$- \frac{F(s)}{(V \cdot A)^4} r_j + \frac{F(s)}{(V \cdot A)^3} \Delta_j$$

$$- \frac{F'(s)}{(V \cdot A)^4} r_j - \frac{F(s)}{(V \cdot A)^4} r_j - \frac{F(s)}{(V \cdot A)^3} r_j$$

$$- \frac{3F(s)}{(V \cdot A)^5} r_j + \frac{3F(s)}{(V \cdot A)^5} r_j + \frac{3F(s)}{(V \cdot A)^4} r_j$$

$$\begin{aligned}
 & + \frac{F(s)(A \cdot r)}{(r \cdot A)^4} V_i r_j - \frac{F(s)(A \cdot r)}{(r \cdot A)^3} \Delta_{ij} \\
 & - \frac{F(s)}{(r \cdot A)^3} V_i r_j - \frac{2F(s)}{(r \cdot A)^4} V_i r_j + \frac{2F(s)(A \cdot r)}{(r \cdot A)^4} V_i r_j + \frac{2F(s)}{(r \cdot A)^3} V_i r_j \\
 & - \frac{F(s)}{(r \cdot A)^3} A_i r_j
 \end{aligned}$$

$$\Delta_{ij} r_i r_j = 0$$

$$\Delta_{ij} \Delta_{ij} = +4$$

$$\Delta_{ij} r_i r_j = (r \cdot r)$$

$$\Delta_{ij} r_i A_j = (A \cdot r)$$

$$\Delta_{ij} B_i r_j = (B \cdot r)$$

$$\Delta_{ij} V_i V_j = 1$$

□ Φ

$$\begin{aligned}
 & - \frac{2F(s)}{(V \cdot A)^2} - \frac{F(s)}{(V \cdot A)^2} + \frac{4F(s)}{(V \cdot A)^2} - \frac{3F(s)}{(V \cdot A)^3} - \frac{F(s)}{(V \cdot A)^3} + \frac{4F(s)}{(V \cdot A)^3} \\
 & - \frac{F(s)(A \cdot A)}{(V \cdot A)^3} + \frac{3F(s)(A \cdot A)}{(V \cdot A)^3} + \frac{F(s)(A \cdot A)}{(V \cdot A)^3} - \frac{9F(s)(A \cdot A)}{(V \cdot A)^3} \\
 & - \frac{F(s)}{(V \cdot A)^2} - \frac{2F(s)}{(V \cdot A)^3} + \frac{2F(s)(A \cdot A)}{(V \cdot A)^3} + \frac{8F(s)}{(V \cdot A)^3}
 \end{aligned}$$

$$- \frac{F(s)(A \cdot A)}{(V \cdot A)^3}$$

$$\frac{F'(s)}{(s^2+1)^2} \left[-2-1+9-1 \right] + \frac{F(s)}{(s^2+1)^3} \left[-3-1+4-2+2 \right]$$

$$+ \frac{F(s)(s^2+1)}{(s^2+1)^3} \left[-1+3+1-4+2 \right] - 1$$

$$\frac{F(s)}{(s^2+1)^3} \left[-1+3+1-4+2 \right] - 1$$

$$\boxed{\boxed{\Phi=0}}$$

$$\nabla \frac{1}{(r, \hat{r})} = \frac{\hat{r}}{r^3}$$

$$\nabla \frac{1}{(r, \hat{r})} = \frac{1}{(r, \hat{r})^3} \hat{r} - \frac{(\hat{r}, \hat{r})}{(r, \hat{r})^3} \hat{r} = \frac{1}{(r, \hat{r})^2} \hat{r}$$

$$\nabla r_i = -\frac{V_i}{(r, \hat{r})} \hat{r} + \Delta_{ij}$$

$$\frac{\partial}{\partial x^m} \frac{1}{(r, \hat{r})} = \frac{1}{(r, \hat{r})^3} r_m - \frac{(\hat{r}, \hat{r})}{(r, \hat{r})^3} r_m - \frac{1}{(r, \hat{r})^2} V_m$$

$$\frac{\partial}{\partial x^n} V_i = -\frac{1}{(r, \hat{r})} V_i r_n + \Delta_{in}$$

$$\Delta^{mn} \left(\frac{\partial}{\partial x^m} \frac{1}{(r, \hat{r})} \right) \left(\frac{\partial}{\partial x^n} r_i \right) = \left[\frac{1}{(r, \hat{r})^2} V_i + \frac{1}{(r, \hat{r})^3} r_i \right. \\ \left. - \frac{(\hat{r}, \hat{r})}{(r, \hat{r})^3} r_i - \frac{1}{(r, \hat{r})^2} V_i \right]$$

$$h^{ij} = \frac{M[2V^i V^j - \Delta^{ij}]}{(V \cdot A)}$$

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$$h_{jk} = \frac{M[2V_j V_k - \Delta_{jk}]}{(V \cdot A)}$$

$$h^{ij} h_{jk} = \frac{M^2[4V^i V_k - 2V^i V_k - 2V^i V_k + \delta_k^i]}{(V \cdot A)^2}$$

~~$h^{ij} h_{jk}$~~

$$h^{ij} h_{jk} = \frac{M^2 \delta_k^i}{(V \cdot A)^2}$$

$$h^{ij} h_{ji} = \frac{4M^2}{(V \cdot A)^2}$$

$$h^{ij} \frac{\partial h_{jk}}{\partial x^i} + \frac{\partial h^{ij}}{\partial x^i} h_{jk} = M^2 \delta_k^i \frac{\partial}{\partial x^i} \frac{1}{(V \cdot A)^2} = M^2 \frac{\partial}{\partial x^k} \frac{1}{(V \cdot A)^2}$$

$$h^{ij} \frac{\partial h_{ij}}{\partial x^k} + \frac{\partial h^{ij}}{\partial x^k} h_{ij} = 4M^2 \frac{\partial}{\partial x^k} \frac{1}{(V \cdot A)^2}$$

$$h^{ij} \left[\frac{\partial h_{jk}}{\partial x^i} - \frac{\partial h_{ij}}{\partial x^k} \right] + \cancel{h_{jk} \left[\frac{\partial h^{ij}}{\partial x^i} - \frac{\partial h^{ij}}{\partial x^k} \right]} = 0$$

$$\frac{\partial h^{ij}}{\partial x^k} h_{ji} + \frac{\partial h^{ij}}{\partial x^i} h_{jk} = -3M^2 \frac{\partial}{\partial x^k} \frac{1}{(V \cdot A)^2}$$

$$(\vec{V} \cdot \vec{A}) h^{ij} h_{jk} = \frac{M^2 \delta_k^i}{(\vec{V} \cdot \vec{A})}$$

$$\frac{\partial h_{ij}}{\partial x^k} \rightarrow \frac{\partial h_{jk}}{\partial x^i}$$

$$G_k = \Delta^{ij} \frac{\partial}{\partial x^i} h_{jk} = \cancel{h_k^i} \frac{\partial}{\partial x^i} h_k^i$$

$$H_k = \frac{\partial}{\partial x^k} \Delta^{ij} h_{ji} = \frac{\partial}{\partial x^k} h_i^i$$

$$G_k - \frac{1}{2} H_k = \frac{\partial}{\partial x^i} h_k^i - \frac{1}{2} \frac{\partial}{\partial x^k} h_i^i$$

$$G_k - \frac{1}{2} H_k =$$

0

$$\begin{array}{cccc} h_{11} - \lambda & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} - \lambda & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} - \lambda & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} - \lambda \end{array}$$

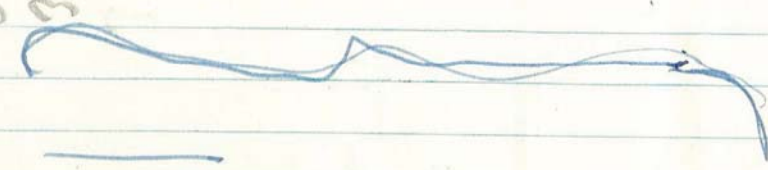
$$\begin{vmatrix}
 h_{11} & h_{12} & h_{13} & h_{14} \\
 h_{21} & h_{22}-\lambda & h_{23} & h_{24} \\
 h_{31} & h_{32} & h_{33}-\lambda & h_{34} \\
 h_{41} & h_{42} & h_{43} & h_{44}-\lambda
 \end{vmatrix} = 0$$

$$\begin{vmatrix}
 h_{11}-\lambda & h_{12} \\
 h_{21} & h_{22}-\lambda
 \end{vmatrix} = \begin{vmatrix}
 h_{11} & h_{12} \\
 h_{21} & h_{22}-\lambda
 \end{vmatrix} - \lambda \begin{vmatrix}
 h_{12} \\
 h_{21}
 \end{vmatrix}$$

$$\begin{vmatrix}
 h_{11}-\lambda & h_{12} \\
 h_{21} & h_{22}-\lambda
 \end{vmatrix} =$$

10 9 8 7 6 5 4 3 2 1
 || || || || || || || || || ||
 10 9 8 7 6 5 4 3 2 1
 + + + + + + + + + +
 3 3 3 3 3 3 3 3 3 3

0



$$\begin{aligned}
 & \left(\begin{array}{c} h_{11} - \lambda \\ h_{21} \\ h_{31} \\ h_{41} \end{array} \right) \cdot \left(\begin{array}{c} h_{12} \\ h_{22} - \lambda \\ h_{32} \\ h_{42} \end{array} \right) - \left(\begin{array}{c} h_{13} \\ h_{23} \\ h_{33} - \lambda \\ h_{43} \end{array} \right) \cdot \left(\begin{array}{c} h_{14} \\ h_{24} \\ h_{34} \\ h_{44} - \lambda \end{array} \right) \\
 & + \left(\begin{array}{c} h_{11} - \lambda \\ h_{21} \\ h_{31} \\ h_{41} \end{array} \right) \cdot \left(\begin{array}{c} h_{12} \\ h_{22} - \lambda \\ h_{32} \\ h_{42} \end{array} \right) - \left(\begin{array}{c} h_{13} \\ h_{23} \\ h_{33} - \lambda \\ h_{43} \end{array} \right) \cdot \left(\begin{array}{c} h_{14} \\ h_{24} \\ h_{34} \\ h_{44} - \lambda \end{array} \right) \\
 & + \left(\begin{array}{c} h_{11} - \lambda \\ h_{21} \\ h_{31} \\ h_{41} \end{array} \right) \cdot \left(\begin{array}{c} h_{12} \\ h_{22} - \lambda \\ h_{32} \\ h_{42} \end{array} \right) - \left(\begin{array}{c} h_{13} \\ h_{23} \\ h_{33} - \lambda \\ h_{43} \end{array} \right) \cdot \left(\begin{array}{c} h_{14} \\ h_{24} \\ h_{34} \\ h_{44} - \lambda \end{array} \right) \\
 & + \left(\begin{array}{c} h_{11} - \lambda \\ h_{21} \\ h_{31} \\ h_{41} \end{array} \right) \cdot \left(\begin{array}{c} h_{12} \\ h_{22} - \lambda \\ h_{32} \\ h_{42} \end{array} \right) - \left(\begin{array}{c} h_{13} \\ h_{23} \\ h_{33} - \lambda \\ h_{43} \end{array} \right) \cdot \left(\begin{array}{c} h_{14} \\ h_{24} \\ h_{34} \\ h_{44} - \lambda \end{array} \right)
 \end{aligned}$$

new

$$h_{11}h_{22} - h_{12}h_{21} -$$

ECUACION SECULAR

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Einführung in die DETERMINANTEN-
THEORIE. Gerhard Kowalewski.
Chelsea Publishing Co. 1948. (New York)

Data:

$$D(x) = \begin{vmatrix} a_{11}+x & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22}+x & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33}+x & \cdots & a_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn}+x \end{vmatrix}$$

Desarrollese $D(x)$ según potencias de x .

$$D(x) = \begin{vmatrix} a_{11}+x & a_{12}+0 & a_{13}+0 & \cdots & a_{1n}+0 \\ a_{21}+0 & a_{22}+x & a_{23}+0 & \cdots & a_{2n}+0 \\ a_{31}+0 & a_{32}+0 & a_{33}+x & \cdots & a_{3n}+0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1}+0 & a_{n2}+0 & a_{n3}+0 & \cdots & a_{nn}+x \end{vmatrix}$$

$D(x) = \sum$ de 2^n determinantes,
~~según~~

$$D(x) =$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{vmatrix} +$$

$$X \left[\begin{vmatrix} a_{22} & a_{23} & \dots & a_{2n} \\ a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots \\ a_{n2} & a_{n3} & \dots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} & \dots & a_{1n} \\ a_{31} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n3} & \dots & a_{nn} \end{vmatrix} \right]$$

$$+ \begin{vmatrix} a_{11} & a_{12} & a_{14} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{24} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{34} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n4} & \dots & a_{nn} \end{vmatrix} + \dots$$

+ x²

$$\begin{vmatrix} a_{33} & a_{34} & \cdots & a_{3n} \\ a_{43} & a_{44} & \cdots & a_{4n} \\ & & & \\ a_{n3} & a_{n4} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{12} & a_{24} & \cdots & a_{2n} \\ a_{42} & a_{44} & \cdots & a_{4n} \\ & & & \\ a_{n2} & a_{n4} & \cdots & a_{nn} \end{vmatrix} \quad 114$$

+ - - -

$$= D(x)$$

$$\begin{vmatrix} h_{11}+x & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22}+x & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33}+x & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44}+x \end{vmatrix}$$

$$D(x) = \begin{vmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{vmatrix} + \begin{vmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{vmatrix} + \begin{vmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{vmatrix} + \begin{vmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{vmatrix}$$

Si h_{ij} es un tensor doblemente covariante y se somete a una transformación restringida de Lorentz ¿qué ocurre con los coeficientes del desarrollo de $D(x)$?

$$h'_{ij} = L_i^p L_j^q h_{pq}$$

$$x'^i = L_p^i x^p$$

$$x^i = L_p^i x^p$$

$$L_j^i L_k^j = \delta_k^i$$

$$|L_j^i| = 1$$

$$|h'_{ij}| = |L_i^p L_j^q h_{pq}| = |h_{pq}|$$

$$\begin{pmatrix} L_1^1 & L_2^1 & L_3^1 & L_4^1 \\ L_1^2 & L_2^2 & L_3^2 & L_4^2 \\ L_1^3 & L_2^3 & L_3^3 & L_4^3 \\ L_1^4 & L_2^4 & L_3^4 & L_4^4 \end{pmatrix} = \begin{pmatrix} l_1^1 - l_1^2 & -l_1^3 & -l_1^4 \\ -l_2^1 & l_2^2 & l_2^3 & +l_2^4 \\ -l_3^1 & l_3^2 & l_3^3 & +l_3^4 \\ -l_4^1 & +l_4^2 & +l_4^3 & l_4^4 \end{pmatrix}$$

$$|l_j^i| = +1$$

$$|L_j^i| = |l_j^i|$$

$$|l_j^i|^2 = 1$$

h

$$\Delta_{ij} x^i x^j = \Delta_{pq} x^p x^q$$

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$$\Delta_{ij} l_p^i x^p l_q^j x^q = \Delta_{pq} x^p x^q$$

$$(\Delta_{ij} l_p^i l_q^j) = \Delta_{pq}$$

$$\begin{pmatrix} l_1^1 & l_2^1 & l_3^1 & l_4^1 \\ l_1^2 & l_2^2 & l_3^2 & l_4^2 \\ l_1^3 & l_2^3 & l_3^3 & l_4^3 \\ l_1^4 & l_2^4 & l_3^4 & l_4^4 \end{pmatrix} = \begin{pmatrix} L_1^1 & L_2^1 & L_3^1 & L_4^1 \\ L_1^2 & L_2^2 & L_3^2 & L_4^2 \\ L_1^3 & L_2^3 & L_3^3 & L_4^3 \\ L_1^4 & L_2^4 & L_3^4 & L_4^4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

El determinante $|h_{ij}|$ es invariante¹¹⁸
en las transformaciones restringidas
de Lorentz.

También es invariante h_j^i

$$h_j'^i = l_p^i L_j^q h_q^p$$

$$|h_j'^i| = |l_p^i| |L_j^q| |h_q^p| = |h_q^p|$$

$$h_1^1 + h_2^2 + h_3^3 + h_4^4 = \left\{ \begin{array}{l} \text{traza del} \\ \text{tensor} \end{array} \right.$$

$$h_1^1 + h_2^2 + h_3^3 + h_4^4 \quad \underline{\underline{\text{invariante}}}$$

$$\varepsilon_{ijkl} h_a^i h_b^j h_c^k h_d^l = \varepsilon_{abcd}$$

$$\begin{vmatrix} h_2^{1/2} & h_3^{1/2} & h_4^{1/2} \\ h_2^{1/3} & h_3^{1/3} & h_4^{1/3} \\ h_2^{1/4} & h_3^{1/4} & h_4^{1/4} \end{vmatrix} =$$

$$\begin{vmatrix} l_p^2 L_2^q h_q^p & l_p^2 L_3^q h_q^p & l_p^2 L_4^q h_q^p \\ l_p^3 L_2^q h_q^p & l_p^3 L_3^q h_q^p & l_p^3 L_4^q h_q^p \\ l_p^4 L_2^q h_q^p & l_p^4 L_3^q h_q^p & l_p^4 L_4^q h_q^p \end{vmatrix}$$

Cofactor de ~~h_j^i~~ es H_i^j



$$h_1^1 H_1^1 + h_2^2 H_2^2 + \dots$$

$$h_{11} H^{11} - h_{22} H^{22} - h_{33} H^{33} - h_{44} H^{44}$$

$$\hat{r} \cdot \nabla h_{jk} = -h_{jk}$$

$$a_i = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) v^j v^k$$

$$\hat{r} \cdot \nabla h_{jk} + \hat{r} \cdot \nabla \frac{dh_{jk}}{ds} = - \frac{dh_{jk}}{ds}$$

$$\hat{a} = \hat{r} \cdot \nabla h_{jk}$$

$$\hat{a} = (\hat{r} \cdot \nabla h) \hat{r} - \nabla \{ (h \hat{r}) \cdot \hat{r} \}$$

$$\hat{r} \cdot \hat{a} = \hat{r} \cdot (\hat{r} \cdot \nabla h) \hat{r} + (h \hat{r}) \cdot \hat{r}$$

$$\hat{r} \cdot \frac{d}{ds} (\hat{r} \cdot \hat{r}) = 1 + (\hat{r} \cdot \hat{a})$$

$$\frac{d}{ds} (\hat{r} \cdot \hat{r}) = \left[1 + \left[(h \hat{r}) \cdot \hat{r} \right] + \hat{r} \cdot \left[(\hat{r} \cdot \nabla h) \hat{r} \right] \right]$$